Ψηφιακή Επεξεργασία Εικόνας

Αποκατάσταση και ανακατασκευή εικόνας

Διδάσκων: Αναπληρωτής Καθηγητής Νίκου Χριστόφορος

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Digital Image Processing

Image Restoration and Reconstruction
(Linear Restoration Methods)

Christophoros Nikou
cnikou@cs.uoi.gr

University of Ioannina - Department of Computer Science

Contents

In this lecture we will look at linear image restoration techniques

– Differentiation of matrices and vectors
– Linear space invariant degradation
– Restoration in absence of noise
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– Restoration in presence of noise
  • Inverse filter
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C. Nikou – Digital Image Processing (E12)
Differentiation of Matrices and Vectors

Notation:

A is a $M \times N$ matrix with elements $a_{ij}$.

x is a $N \times 1$ vector with elements $x_i$.

$f(x)$ is a scalar function of vector x.

g(x) is a $M \times 1$ vector valued function of vector x.

Scalar derivative of a matrix.

$A$ is a $M \times N$ matrix with elements $a_{ij}$.

\[
\frac{\partial A}{\partial t} = \begin{pmatrix}
\frac{\partial a_{11}}{\partial t} & \cdots & \frac{\partial a_{1N}}{\partial t} \\
\vdots & \ddots & \vdots \\
\frac{\partial a_{M1}}{\partial t} & \cdots & \frac{\partial a_{MN}}{\partial t}
\end{pmatrix}
\]
Vector derivative of a function (gradient).

- $\mathbf{x}$ is a $N \times 1$ vector with elements $x_i$.
- $f(\mathbf{x})$ is a scalar function of vector $\mathbf{x}$.

\[
\frac{\partial f}{\partial \mathbf{x}} = \nabla f = \left( \frac{\partial f}{\partial x_1} \ldots \frac{\partial f}{\partial x_N} \right)^T
\]

Vector derivative of a vector (Jacobian):

- $\mathbf{x}$ is a $N \times 1$ vector with elements $x_i$.
- $g(\mathbf{x})$ is a $M \times 1$ vector valued function of vector $\mathbf{x}$.

\[
\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \left( \begin{array}{c} \frac{\partial g_1}{\partial x_1} \ldots \frac{\partial g_1}{\partial x_N} \\ \vdots \ldots \vdots \\ \frac{\partial g_M}{\partial x_1} \ldots \frac{\partial g_M}{\partial x_N} \end{array} \right)
\]
Some useful derivatives.

\(x\) is a \(N\times1\) vector with elements \(x_i\).

\(b\) is a \(N\times1\) vector with elements \(b_i\).

\[
\frac{\partial}{\partial x} (b^T x) = b
\]

It is the derivative of the scalar valued function \(b^T x\) with respect to vector \(x\).

If \(A\) is symmetric:

\[
\frac{\partial}{\partial x} (x^T Ax) = (A + A^T) x
\]
Differentiation of Matrices and Vectors (cont...)

Some useful derivatives.

\( x \) is a \( N \times 1 \) vector with elements \( x_i \).

\( b \) is a \( M \times 1 \) vector with elements \( b_i \).

\( A \) is a \( M \times N \) matrix with elements \( a_{ij} \).

\[
\frac{\partial}{\partial x} \|Ax + b\|^2 = 2A^T (Ax + b)
\]

It may be proved using the previous properties.

Linear, Position-Invariant Degradation

We now consider a degraded image to be modelled by:

\[
g(x, y) = h(x, y) * f(x, y) + \eta(x, y)
\]

where \( h(x, y) \) is the impulse response of the degradation function (i.e. point spread function blurring the image).

The convolution implies that the degradation mechanism is linear and position invariant (it depends only on image values and not on location).
In the Fourier domain:

\[ G(k, l) = H(k, l)F(k, l) + N(k, l) \]

where multiplication is element-wise.

In matrix-vector form:

\[ g = Hf + \eta \]

where \( H \) is a doubly block circulant matrix and \( f, g, \) and \( \eta \) are vectors (lexicographic ordering).

If the degradation function is unknown the problem of simultaneously recovering \( f(x, y) \) and \( h(x, y) \) is called blind deconvolution.
Estimating the point spread function

• In what follows, we consider that the degradation function is known.

• If the psf is not known, some basic methods to estimate it are:
  – By observation
    • Apply sharpening filters to a sub-image $g_{(m,n)}$ where the signal is strong (there is almost no noise) and obtain a visually pleasant result $f_{(m,n)}$. The psf may be approximated by $H_{(k,l)} = G_{(k,l)} \cdot F_{(k,l)}$.
    • The task needs trial and error and may be tedious.
    • Used in special circumstances (e.g. Restoration of old photographs)

Estimating the point spread function (cont.)

• If the psf is not known, some basic methods to estimate it are:
  – By experimentation
    • If the acquisition equipment or a similar one is available an image similar to the degraded may be obtained by varying the system settings.
    • Then obtain the image of an impulse (small dot of light) using the same settings.
Estimating the point spread function (cont.)

• If the psf is not known, some \textbf{basic} methods to estimate it are:
  – By modeling: \textit{atmospheric turbulence}:

\[ H(u, v) = \exp\left(-k \left(u^2 + v^2\right)^{\frac{1}{2}}\right) \]

Estimating the point spread function (cont.)

• By modeling: \textbf{planar motion}
  – \( x_0(t) \) and \( y_0(t) \) are the time varying components of motion at each pixel.
  – The total exposure at any pixel is obtained by integrating the instantaneous exposure over the time the shutter is open.
  – Assumption: the shutter opening and closing is instantaneous.
  – If \( T \) is the duration of the exposure, the recorded image is expressed by:

\[ g(x, y) = \int_{0}^{T} f[x - x_0(t), y - y_0(t)] dt \]
Estimating the point spread function (cont.)

\[
G(u, v) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x, y) e^{-j2\pi(ux + vy)} \, dx \, dy
\]

\[
= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ f[x - x_0(t), y - y_0(t)] dt \right] e^{-j2\pi(ux + vy)} \, dx \, dy
\]

\[
= \int_0^T \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f[x - x_0(t), y - y_0(t)] e^{-j2\pi(ux + vy)} \, dx \, dy \right] dt
\]

\[
= \int_0^T F(u, v)e^{-j2\pi[aux_0(t) + by_0(t)]} dt
\]

\[
= F(u, v)\int_0^T e^{-j2\pi[aux_0(t) + by_0(t)]} dt \Leftrightarrow
\]

\[
H(u, v) = \int_0^T e^{-j2\pi[aux_0(t) + by_0(t)]} dt
\]

Estimating the point spread function (cont.)

- Considering uniform linear motion:

\[
x_0(t) = a \frac{t}{T}, \quad y_0(t) = b \frac{t}{T}
\]

- The psf becomes:

\[
H(u, v) = \int_0^T e^{-j2\pi(aut + vbt)/T} dt
\]

\[
= \frac{T}{\pi(ua + vb)} \sin \left[ \pi \left( ua + vb \right) \right] e^{-j\pi(ua + vb)}
\]
Estimating the point spread function (cont.)

• Result of blurring with:

\[ x_0(t) = a \frac{t}{T}, \quad y_0(t) = b \frac{t}{T}, \quad a = b = 0.1, \quad T = 1 \]

Linear Restoration

Using the imaging system

\[ g = Hf + \eta \]

we want to estimate the true image from the degraded observation with known degradation \( H \).

A linear method applies an operator (a matrix) \( P \) to the observation \( g \) to estimate the unobserved noise-free image \( f \):

\[ \hat{f} = Pg \]
When there is no noise:
\[ g = Hf \]
an obvious solution would be to use the inverse filter:
\[ P = H^{-1} \]
yielding
\[ \hat{f} = Pg = H^{-1}g = H^{-1}Hf = f \]

For a \( N\times N \) image, \( H \) is a \( N^2 \times N^2 \) matrix!
To tackle the problem we transform it to the Fourier domain.
\( H \) is doubly block circulant and therefore it may be diagonalized by the 2D DFT matrix \( W \):
\[ H = W^{-1} \Lambda W \]
Restoration in Absence of Noise
The Inverse Filter (cont...)

\[ H = W^{-1} \Lambda W \]

where
\[ \Lambda = \text{diag}\{H(1,1),...,H(N,1),H(1,2),...H(N,N)\} \]

Therefore:
\[ \hat{f} = P g \iff \hat{f} = H^{-1} g \iff \hat{f} = (W^{-1} \Lambda W)^{-1} g \]
\[ \iff \hat{f} = W^{-1} \Lambda^{-1} W g \iff W \hat{f} = WW^{-1} \Lambda^{-1} W g \]
\[ \iff \hat{F} = \Lambda^{-1} G \]

This is the vectorized form of the DFT of the image:
\[ \hat{F} = \Lambda^{-1} G \iff \hat{F}(k,l) = \frac{G(k,l)}{H(k,l)} \]

Take the inverse DFT and obtain \( f(m,n) \).
Problem: what happens if \( H(k,l) \) has zero values?
Cannot perform inverse filtering!
A solution is to set:

\[
\hat{F}(k,l) = \begin{cases} 
G(k,l) & , \ H(k,l) \neq 0 \\
H(k,l) & , \ H(k,l) = 0
\end{cases}
\]

which is a type of pseudo-inversion.

Notice that the signal cannot be restored at locations where \(H(k,l)=0\).

A pseudo-inverse filter also arises by the unconstrained least squares approach.

Find the image \(f\), that, when it is blurred by \(H\), it will provide an observation as close as possible to \(g\), i.e. It minimizes the distance between \(Hf\) and \(g\).
Restoration in Absence of Noise
The Pseudo-inverse Filter (cont...)

This distance is expressed by the norm:

\[ J(f) = \|Hf - g\|^2 \]

\[ \min_f \{ J(f) \} \iff \frac{\partial J}{\partial f} = 0 \iff \frac{\partial}{\partial f} \left( \|Hf - g\|^2 \right) = 0 \]

\[ \iff 2H^T (Hf - g) = 0 \iff 2H^T Hf = 2H^T g \]

\[ \iff f = \left( H^T H \right)^{-1} H^T g \]

---

Restoration in Presence of Noise
The Inverse Filter

Recall the imaging model with spatially invariant degradation and noise

\[ g(x, y) = h(x, y) * f(x, y) + \eta(x, y) \]

\[ G(k, l) = H(k, l) F(k, l) + N(k, l) \]

\[ g = Hf + \eta \]
### Restoration in Presence of Noise
#### The Inverse Filter (cont...)

Applying the inverse filter in the Fourier domain:

\[
G(k, l) = H(k, l)F(k, l) + N(k, l)
\]

\[
\Leftrightarrow \hat{F}(k, l) = F(k, l) + \frac{N(k, l)}{H(k, l)}
\]

Even if we know \(H(k, l)\) we cannot recover \(F(k, l)\) due to the second term.

If \(H(k, l)\) has small values the second term dominates (it goes to infinity if \(H(k, l) = 0\)).

---

### Restoration in Presence of Noise
#### The Inverse Filter (cont...)

- One approach to get around the problem is to limit the ratio \(G(k, l) / H(k, l)\) to frequencies near the origin that have lower probability of being zero.
- We know that \(H(0, 0)\) is usually the highest value of the DFT.
- Thus, by limiting the analysis to frequencies near the origin we reduce the probability of encountering zero values.
### Restoration in Presence of Noise
#### The Inverse Filter (cont...)

**Blurring degradation**

![Image of blurring degradation](image)

**Inverse filter with cut-off**

![Image of inverse filter with cut-off](image)
Restoration in Presence of Noise  

Wiener Filter

- So far we assumed nothing about the statistical properties of the image and noise.
- We now consider image and noise as random variables and the objective is to find an estimate of the uncorrupted image $f$ such that the mean square error between the estimate and the image is minimized:

$$
\min_{\hat{f}} \left\{ E \left[ (f - \hat{f})^2 \right] \right\}
$$

where $E[x]$ is the expected value of vector $x$.

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<tr>
<td><strong>Recall also the definition of the correlation matrix between two vectors $x$ and $y$:</strong></td>
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</table>
| $R_{xy} = E[x y^T] = \begin{bmatrix}
E[x_1 y_1] & E[x_1 y_2] & \cdots & E[x_1 y_N] \\
\vdots & \vdots & \ddots & \vdots \\
E[x_N y_1] & E[x_N y_2] & \cdots & E[x_N y_N]
\end{bmatrix}$ |
| **We assume that the image and the noise are uncorrelated:** |
| $R_{nf} = R_{fn} = 0$ |
Restoration in Presence of Noise
Wiener Filter (cont...)

• We are looking for the best estimate:

\[
\min_{\hat{f}} \left\{ E \left[ (f - \hat{f})^2 \right] \right\}
\]

• Let’s confine our estimate to be obtainable by a linear operator on the observation:

\[
\hat{f} = Pg
\]

and the goal is to find the best matrix \( P \).

\[ J(\hat{f}) = E\left[ (f - \hat{f})^2 \right] = E\left[ \|f - \hat{f}\|^2 \right] = E\left[ \|f - Pg\|^2 \right] \]

Denoting by \( p_n^T \) the \( n \)-th row of \( P \):

\[ J(\hat{f}) = E\left[ \sum_n (f_n - p_n^T g)^2 \right] = \sum_n E\left[ (f_n - p_n^T g)^2 \right] \]
Restoration in Presence of Noise
Wiener Filter (cont...)

\[
J(\hat{f}) = \sum_n E\left[ (f_n - p_n^T g)(f_n - p_n^T g)^T \right]
\]

\[
= \sum_n E\left[ f_n f_n^T - p_n^T g f_n^T - f_n^T g p_n + p_n^T g g^T p_n \right]
\]

\[
= \sum_n E\left[ f_n f_n^T \right] - p_n^T E\left[ g f_n^T \right] - E\left[ f_n g^T \right] p_n + p_n^T E\left[ g g^T \right] p_n
\]

\[
= \sum_n R_{f_n f_n} - 2p_n^T R_{g f_n} + p_n^T R_{g g} p_n
\]

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Restoration in Presence of Noise
Wiener Filter (cont...)

• We can now minimize the sum with respect to each term:

\[
\frac{\partial}{\partial p_n} \left( R_{f_n f_n} - 2p_n^T R_{g f_n} + p_n^T R_{g g} p_n \right) = 0
\]

\[
\iff -2R_{g f_n} + 2R_{g g} p_n = 0 \iff p_n = R_{g g}^{-1} R_{g f_n}
\]

\[
\iff p_n^T = R_{f g} R_{g g}^{-1}
\]

C. Nikou – Digital Image Processing (E12)
Restoration in Presence of Noise
Wiener Filter (cont...)

- Assembling the rows together:
  \[ P = R_{fg} R^{-1}_{gg} \]

- We have to compute the two matrices:
  \[ R_{gg} = E[gg^T] = E[(Hf + \eta)(Hf + \eta)^T] \]
  \[ = E[Hff^T H^T + Hf\eta^T + \eta^T H^T + \eta\eta^T] \]
  \[ = HR_{ff} H^T + HR_{f\eta} + R_{\eta f} H^T + R_{\eta\eta} \]

Assuming noise is uncorrelated with image:
\[ R_{gg} = HR_{ff} H^T + R_{\eta\eta} \]

Also,
\[ R_{fg} = E[fg^T] = E[f (Hf + \eta)^T] = ... = R_{ff} H^T \]

Finally the matrix we are looking for is
\[ P = R_{fg} R^{-1}_{gg} = R_{ff} H^T \left( HR_{ff} H^T + R_{\eta\eta} \right)^{-1} \]
Restoration in Presence of Noise

Wiener Filter (cont...)

• The estimated uncorrupted image is

\[ \hat{f} = P_g \iff \hat{f} = R_{ff} H^T \left( H R_{ff} H^T + R_{\eta\eta} \right)^{-1} g \]

which may be also expressed as

\[ \hat{f} = \left( H^T R_{\eta\eta}^{-1} H + R_{ff}^{-1} \right)^{-1} H^T R_{\eta\eta}^{-1} g \]

• This result is known as the Wiener filter or the minimum mean square error (MMSE) filter.

Restoration in Presence of Noise

Wiener Filter (cont...)

• Special cases:
  
  – No blur \((H=I, g=f+\eta)\): \[ \hat{f} = R_{ff} \left( R_{ff} + R_{\eta\eta} \right)^{-1} g \]
  
  – No noise \((R_{\eta\eta}=0, g=Hf)\): \[ \hat{f} = H^{-1} g \]
    • This is the inverse filter.

  – No blur, no noise \((H=I, R_{\eta\eta}=0)\): \[ \hat{f} = g \]
    • Do nothing on the observation.
Restoration in Presence of Noise
Wiener Filter (cont...)

- The size of the matrix to be inverted poses difficulties and Wiener filter is implemented in the Fourier domain.
- This occurs when $H$ is doubly block circulant (represents convolution) and the image $f$ and noise $\eta$ are wide-sense stationary (w.s.s).

**Definition of a w.s.s. signal:**
1) $E[f(m,n)] = \mu$, independent of $m,n$.
2) $E[f(m,n) f(k,l)] = r(m-k, n-l)$, independent of location.

Reminder: the inverse DFT complex exponential matrix diagonalizes any circulant matrix:

$$H = W^{-1} A_{\eta} W$$

The columns of $W^{-1}$ are the eigenvectors of any circulant matrix $H$.

The corresponding eigenvalues are the DFT values of the signal producing the circulant matrix.

Remember also that $W^T = W$ and that

$$(W^{-1})^T = W^{-1}$$
We will employ the following relations:

\[ H = W^{-1} \Lambda_H W \]

\[ H^T = \left( W^{-1} \Lambda_H W \right)^T = W \Lambda_H W^{-1} \]

If \( H \) is real:

\[ H^T = \left( H^T \right)^* = \left( W \Lambda_H W^{-1} \right)^* = W^* \Lambda^*_H \left( W^{-1} \right)^* \]

\[ = (NW^{-1}) \Lambda^*_H \frac{1}{N} W = W^{-1} \Lambda^*_H W \]

The Wiener solution is now transformed to the Fourier domain:

\[ \hat{f} = R_{ff} H^T \left( HR_{ff} H^T + R_{\eta\eta} \right)^{-1} g \]

\[ = (W^{-1} \Lambda_n W)(W^{-1} \Lambda_n W)[(W^{-1} \Lambda_n W)(W^{-1} \Lambda_n W)(W^{-1} \Lambda^*_n W) + (W^{-1} \Lambda_{\eta\eta} W)]^{-1} g \]

\[ = W^{-1} \Lambda_n \Lambda^*_n W \left[ W^{-1} (\Lambda_n \Lambda_n \Lambda^*_n + \Lambda_{\eta\eta}) W \right]^{-1} g = W^{-1} \Lambda_n \Lambda^*_n (\Lambda_n \Lambda_n \Lambda^*_n + \Lambda_{\eta\eta})^{-1} Wg \]

\[ \Leftrightarrow W\hat{f} = \Lambda_n \Lambda^*_n (\Lambda_n \Lambda_n \Lambda^*_n + \Lambda_{\eta\eta})^{-1} Wg \]

Notice that the matrices are diagonal.
Restoration in Presence of Noise

Wiener Filter (cont...)

\[ W_\hat{f} = \Lambda_{ff}^* \Lambda_H^* (\Lambda_H \Lambda_{ff} \Lambda_H^* + \Lambda_{\eta\eta})^{-1} Wg \]

\[ F(k,l) = \frac{S_{ff}(k,l)H^*(k,l)}{S_{ff}(k,l)|H(k,l)|^2 + S_{\eta\eta}(k,l)} G(k,l) \]

\[ S_{ff}(k,l) = \text{DFT}(R_{ff}(m,n)) \] is the power spectrum of the image \( f(m,n) \).

\[ S_{\eta\eta}(k,l) = \text{DFT}(R_{\eta\eta}(m,n)) \] is the power spectrum of the noise \( \eta(m,n) \).

If \( S_{ff}(k,l) \) is not zero we may define the Signal to Noise Ratio in the frequency domain:

\[ \text{SNR}(k,l) = \frac{S_{ff}(k,l)}{S_{\eta\eta}(k,l)} \]

and the Wiener filter becomes:

\[ F(k,l) = \frac{H^*(k,l)}{|H(k,l)|^2 + \text{SNR}^{-1}(k,l)} G(k,l) \]
A well known estimate of $S_{ff}(k,l)$ is the periodogram (the ML estimate of $R_{ff}$ when $f$ is assumed Gaussian):

$$S_{ff}(k,l) = \frac{1}{MN}|F(k,l)|^2$$

In practice, as $F(k,l)$ is unknown, we use

$$\hat{S}_{ff}(k,l) = \frac{1}{MN}|G(k,l)|^2$$

**FIGURE 5.28** Comparison of inverse and Wiener filtering (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.
Restoration in Presence of Noise

Wiener Filter (cont...)

- Noise variance one order of magnitude less.
- Noise variance ten orders of magnitude less.

Constrained Least Squares Filter

- When we do not have information on the power spectra the Wiener filter is not optimal.
- Another idea is to introduce a smoothness term in our criterion.
- We define smoothness by the quantity $\|Qf\|^2$
  - $Q$ is a high pass filter operator, e.g. the Laplacian,
  - $Q$ is a doubly block circulant matrix representing convolution.
- We look for smooth solutions minimizing $\|Qf\|^2$
We have the following constrained least squares (CLS) optimization problem:

Minimize $\|Qf\|^2$ subject to $Hf = g$

yielding the Lagrange multiplier minimization of the function:

$$J(f, \lambda) = \|Hf - g\|^2 + \lambda \|Qf\|^2$$

Data fidelity term \quad Smoothness term

Parameter $\lambda$ controls the degree of smoothness:

$\lambda = 0$, $f = \left( H^T H \right)^{-1} H^T g$ \quad pseudo-inverse, ultra rough solution

$\lambda \to \infty$, $f = 0$ \quad ultra smooth solution
In the Fourier domain, the constrained least squares filter becomes:

\[
F(k,l) = \frac{H^*(k,l)}{|H(k,l)|^2 + \lambda |Q(k,l)|^2} G(k,l)
\]

Keep always in mind to zero-pad the images properly.

Low noise: Wiener and CLS generate equal results.
High noise: CLS outperforms Wiener if \( \lambda \) is properly selected.
It is easier to select the scalar value for \( \lambda \) than to approximate the SNR which is seldom constant.
Restoration Performance Measures

Original image $f$ and restored image $\hat{f}$

Mean square error (MSE):

$$\text{MSE} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [f(m,n) - \hat{f}(m,n)]^2$$

Restoration Performance Measures (cont...)

The Signal to Noise Ratio (SNR) considers the difference between the two images as noise:

$$\text{SNR} = \frac{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \hat{f}(m,n)^2}{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [f(m,n) - \hat{f}(m,n)]^2}$$
Τέλος Ενότητας

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• Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στα πλαίσια του εκπαιδευτικού έργου του διδάσκοντα.
• Το έργο «Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Ιωαννίνων» έχει χρηματοδοτηθεί μόνο τη αναδιαμόρφωση του εκπαιδευτικού υλικού.
• Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.
Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.
Έχουν προηγηθεί οι κάτωθι εκδόσεις:
• Έκδοση 1.0 διαθέσιμη εδώ.
  http://ecourse.uoi.gr/course/view.php?id=1126
Σημείωμα Αναφοράς

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