ΠΑΝΕΠΙΣΤΗΜΙΟ ΙΩΑΝΝΙΝΩΝ
ΑΝΟΙΚΤΑ ΑΚΑΔΗΜΑΪΚΑ
ΜΑΘΗΜΑΤΑ

ανοικτάμαθήματα
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Ηλεκτρονικοί Υπολογιστές IV

Η δυναμική ενός μοντέλου Keynsian

Διδάσκων: Επίκουρος Καθηγητής Αθανάσιος Σταυρακούδης

Ευρωπαϊκή Ένωση
Ευρωπαϊκό Κοινωνικό Ταμείο
ΕΠΙΧΕΙΡΗΣΙΑΚΟ ΠΡΟΓΡΑΜΜΑ
ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ & ΘΡΗΣΚΕΥΜΑΤΩΝ, ΠΟΛΙΤΙΣΜΟΥ & ΑΘΛΗΤΙΣΜΟΥ
ΕΙΔΙΚΗ ΥΠΗΡΕΣΙΑ ΔΙΑΧΕΙΡΙΣΗΣ
ΕΣΠΑ 2007-2013
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

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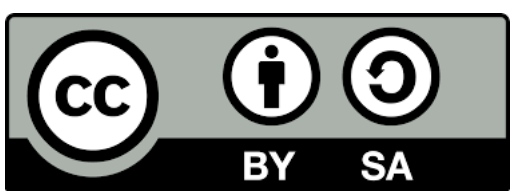
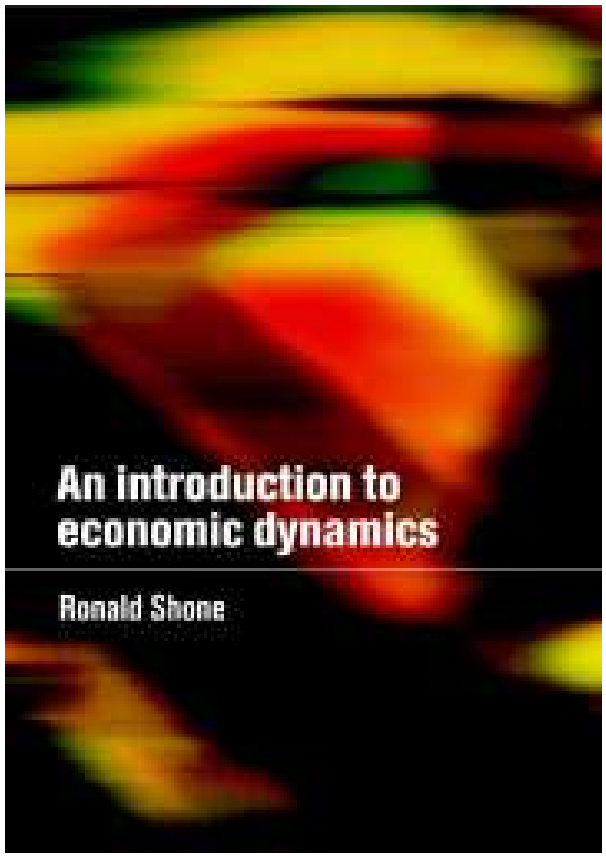


Keynes Dynamics

Athanassios Stavrakoudis

http://stavrakoudis.econ.uoi.gr

Bibliography



Ronald Shone
An Introduction to Economic Dynamics
Cambridge, 2001

Keynes Model

Description

$$\begin{aligned} C &= a + b\,Y \\ E &= C + I + G \\ Y &= E \end{aligned}$$

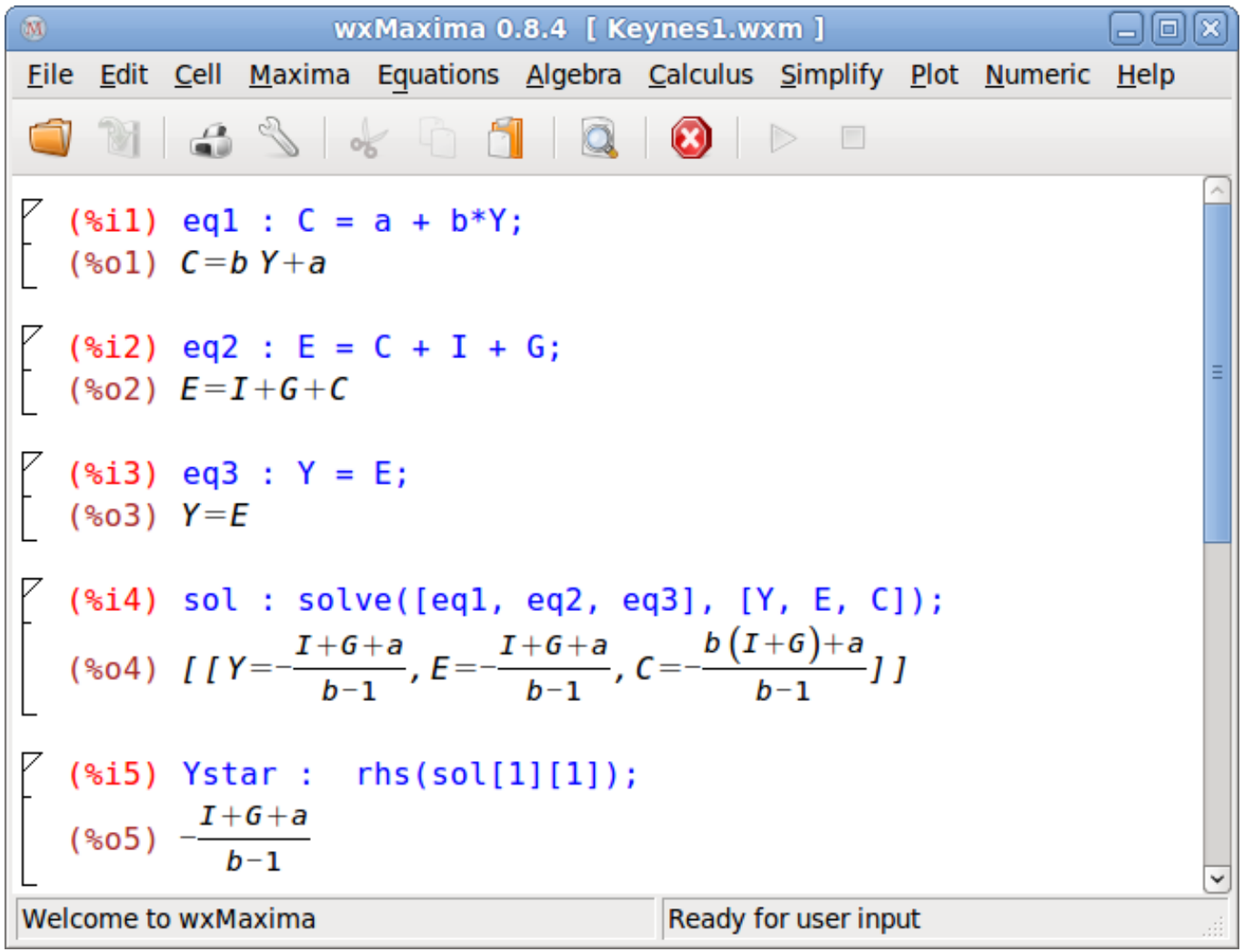
- C = consumption expenditure
- Y = national income
- E = total expenditure
- I = investment expenditure
- G = (government expenditure
- a = autonomous consumption
- b = marginal propensity to consume

Equilibrium

Equilibrium

$$
\begin{aligned}
C &= a + b\,Y \\
E &= C + I + G \\
Y &= E \\
E &= a + b\,Y + I + G \\
Y &= a + b\,Y + I + G \\
Y^* &= \frac{a+I+G}{1-b}
\end{aligned}
$$

Maxima solution



http://stavrakoudis.econ.uoi.gr/stavrakoudis/?iid=534

Multiplier

Equilibrium between two points

$$
\begin{aligned}
Y_1^* &= \frac{a+I_1+G}{1-b} \\
Y_2^* &= \frac{a+I_2+G}{1-b} \\
\Delta Y &= Y_2^* - Y_1^* = \frac{a+I_2+G}{1-b} - \frac{a+I_1+G}{1-b} = \frac{I_2-I_1}{1-b} = \frac{\Delta I}{1-b} \\
k &= \frac{\Delta Y}{\Delta I} = \frac{1}{1-b}
\end{aligned}
$$

Keynes Dynamic System

Dynamics

$$\begin{aligned} C_t &= a + b\,Y_t \\ E_t &= C_t + I + G \\ \Delta Y_{t+1} &= \lambda(E_t - Y_t) \end{aligned}$$

- $E_t - Y_t$, excess demand
- $\lambda > 0$, income adjustment,

Equilibrium

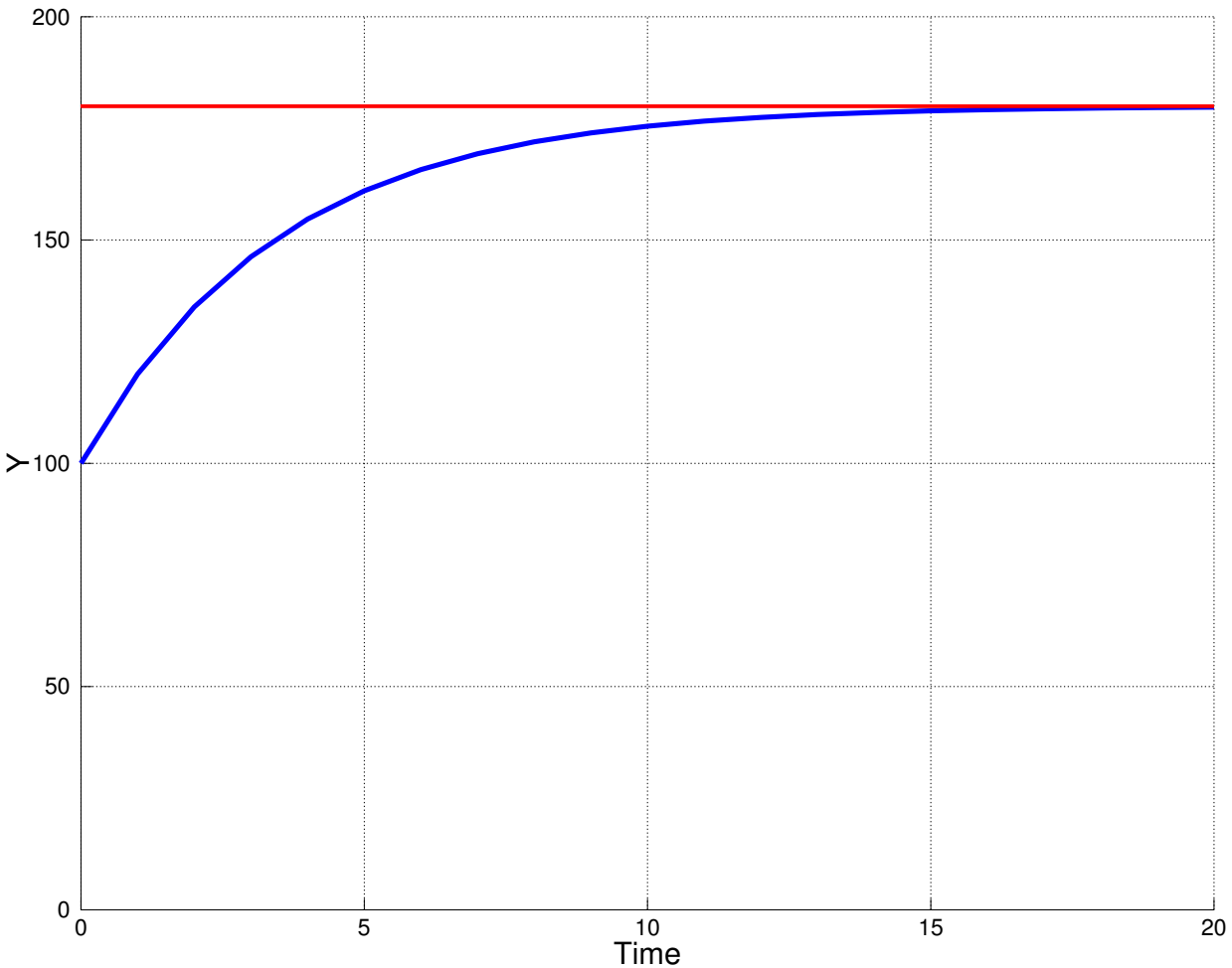
$$\begin{aligned} \Delta Y_{t+1} &= 0 \\ E_t &= Y_t \end{aligned}$$

Octave/Matlab code

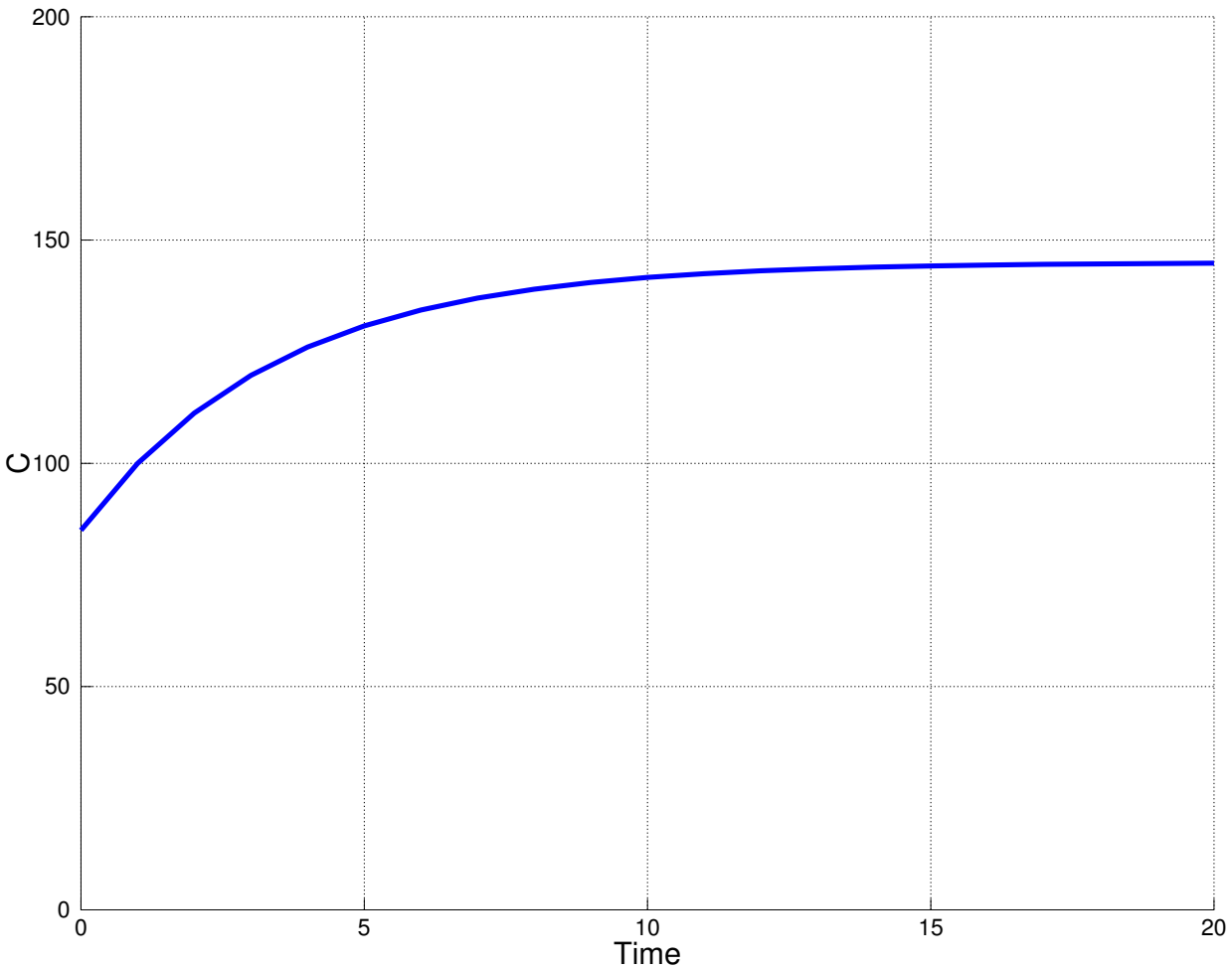
```
1  a      = 10;
2  b      = 0.75;
3  I      = 25;
4  G      = 10;
5  lambda = 1;
6  T      = 20;
7  Y      = zeros(T+1, 1);
8  C      = zeros(T+1, 1);
9  E      = zeros(T+1, 1);
10 dEY    = zeros(T+1, 1);
11
12 Y(1)   = 100;
13 C(1)   = a + b*Y(1);
14 E(1)   = C(1) + I + G;
15 Ystar  = (a+I+G) / (1-b);
16
17 for (t=1:T)
18   Y(t+1) =  lambda * (a+I+G) + (1-lambda*(1-b)) * Y(t);
19 end
20
21 C   = a + b*Y;
22 E   = C + I + G;
23 dEY = E - Y;
```

Keynes1.m

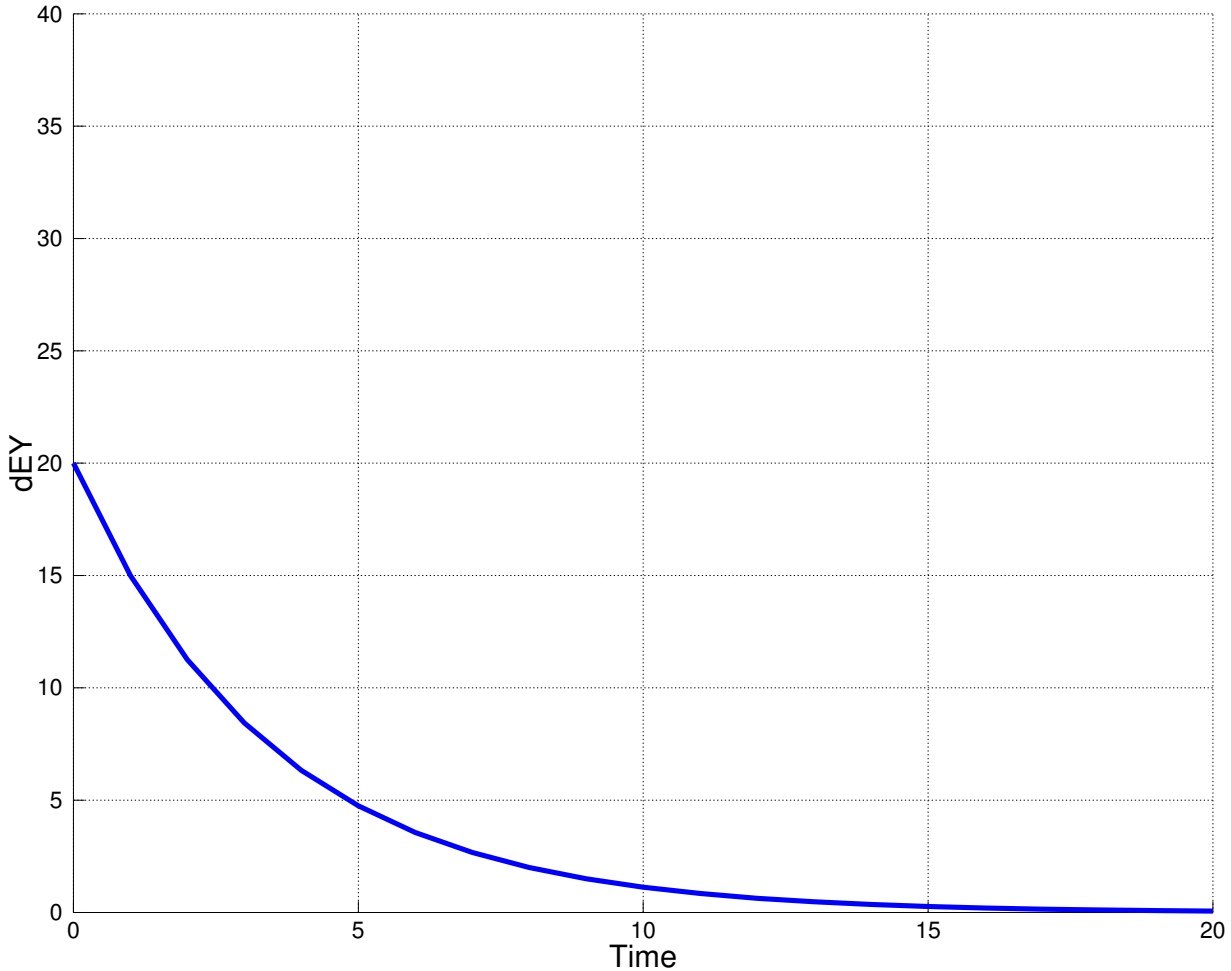
National Income, Y(1)=100



Consumption Expenditure, Y(1)=100



dEY, Y(1)=100

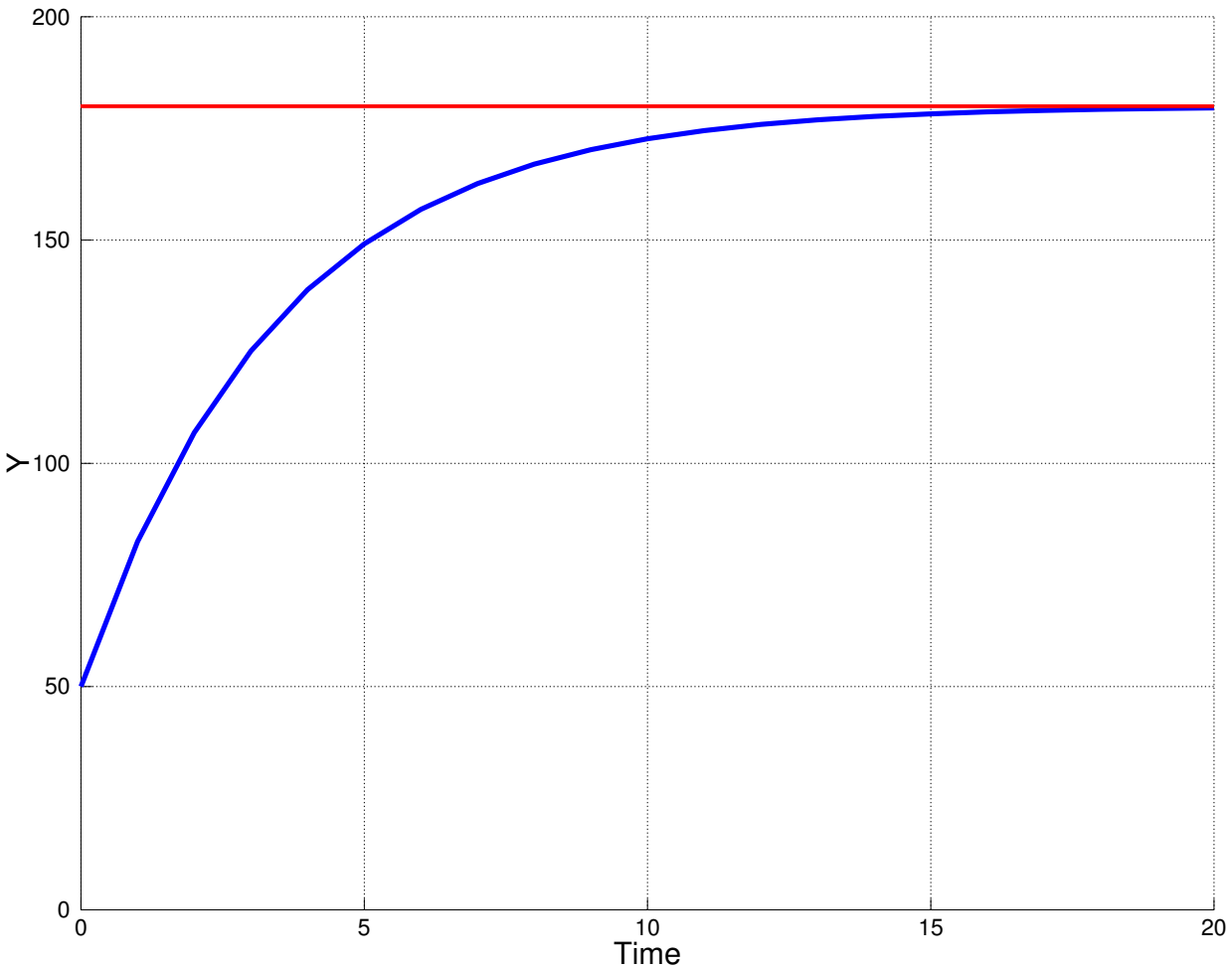


Question

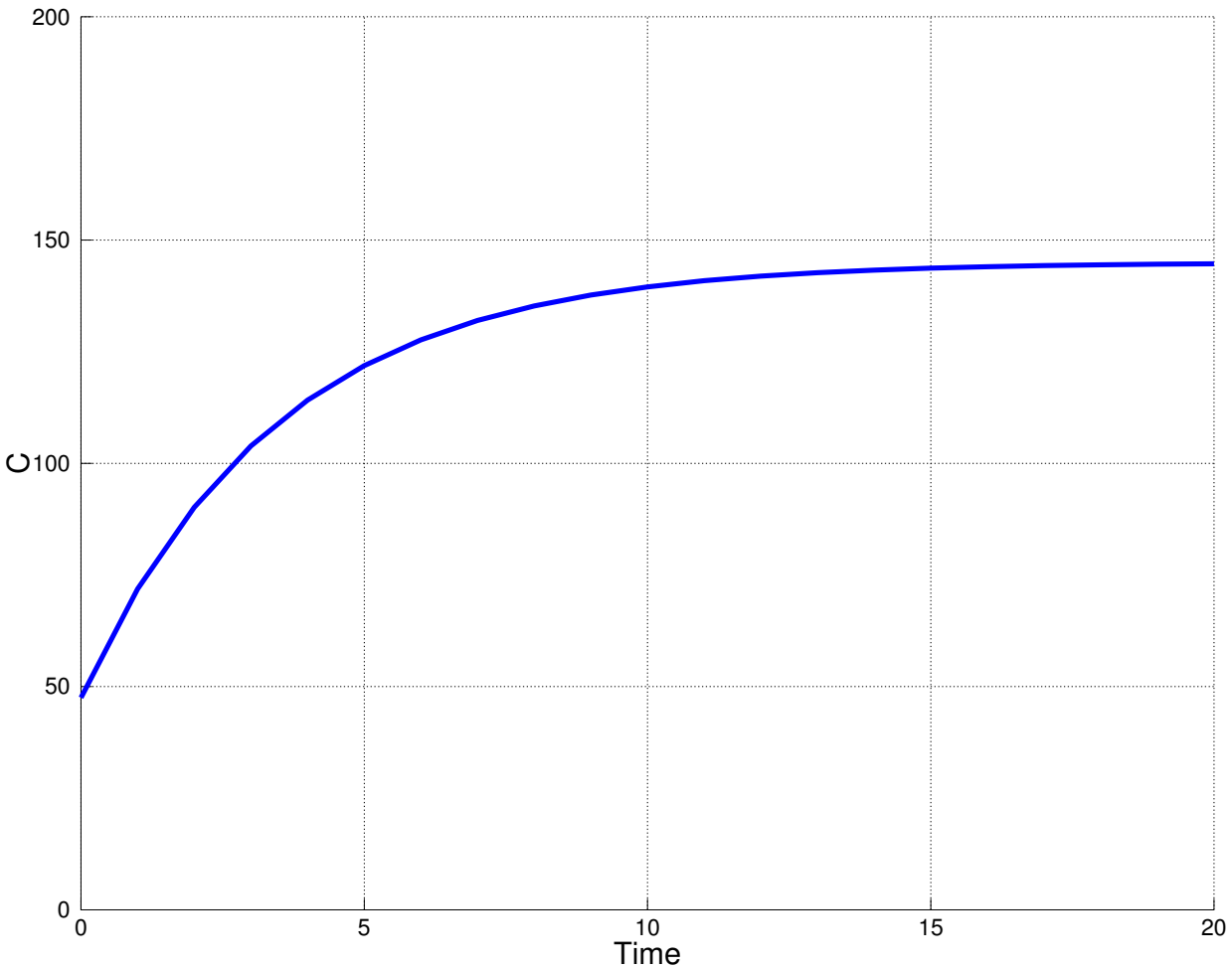
$$\begin{aligned} C_t &= a + b\,Y_t \\ E_t &= C_t + I + G \\ \Delta Y_{t+1} &= \lambda(E_t - Y_t) \end{aligned}$$

- What if we change the initial value of Y?
- Does Y^* change?
- If yes, in what direction?
- Does equilibrium exists for any $Y(0)$?
- Is the value of 180 we found before an attractor or repellor?
- What determines Y^*?

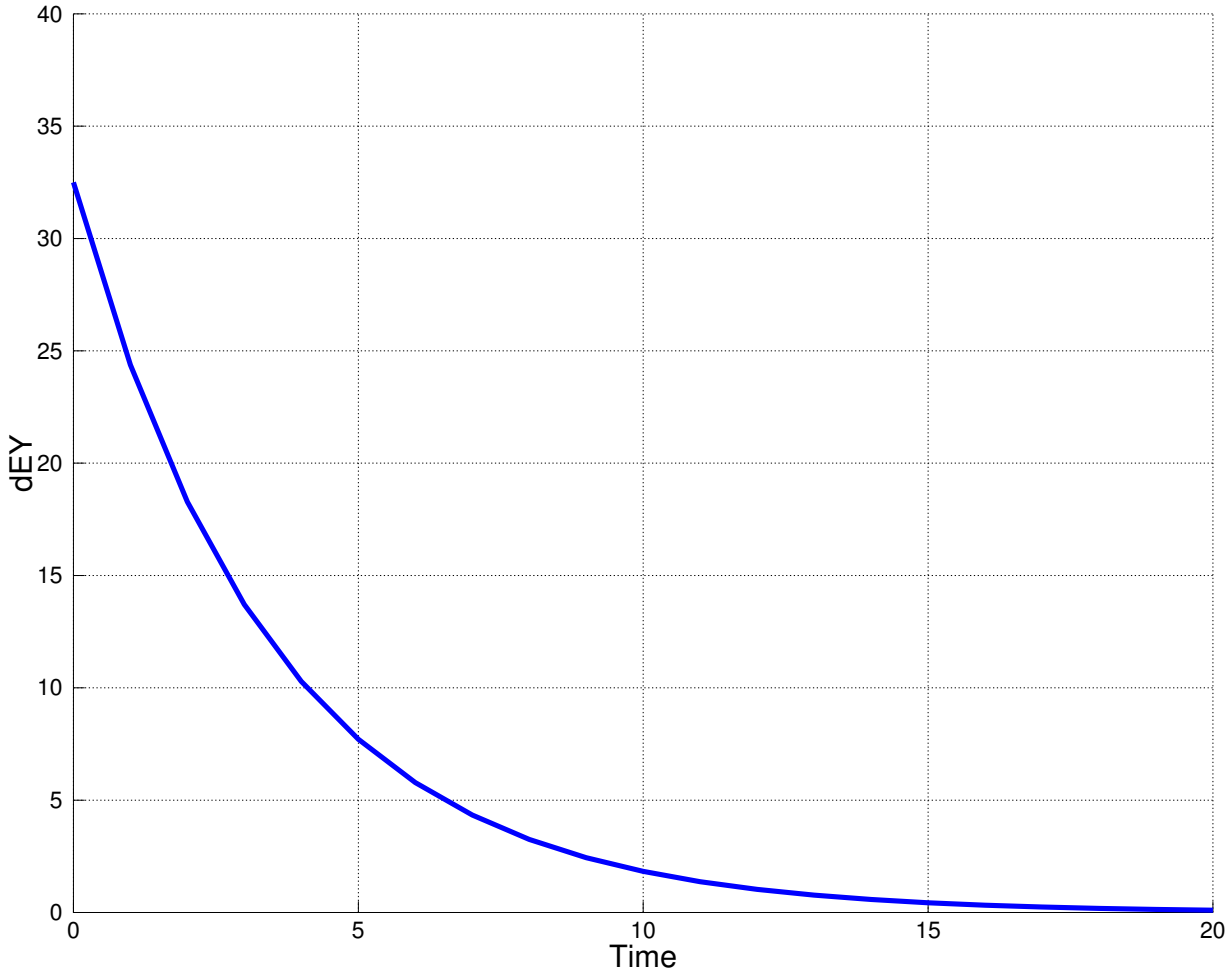
National Income, Y(1)=50



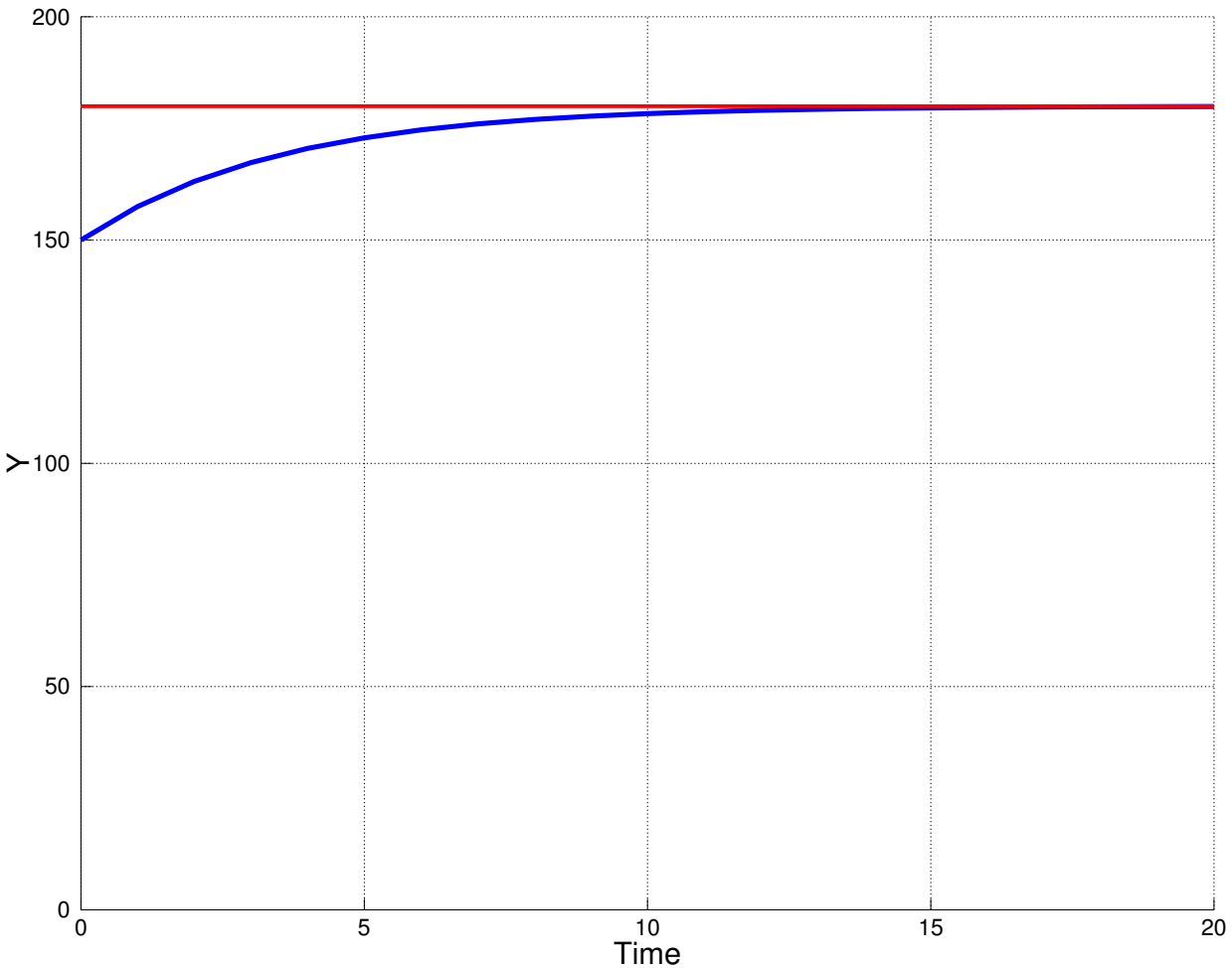
Consumption Expenditure, Y(1)=50



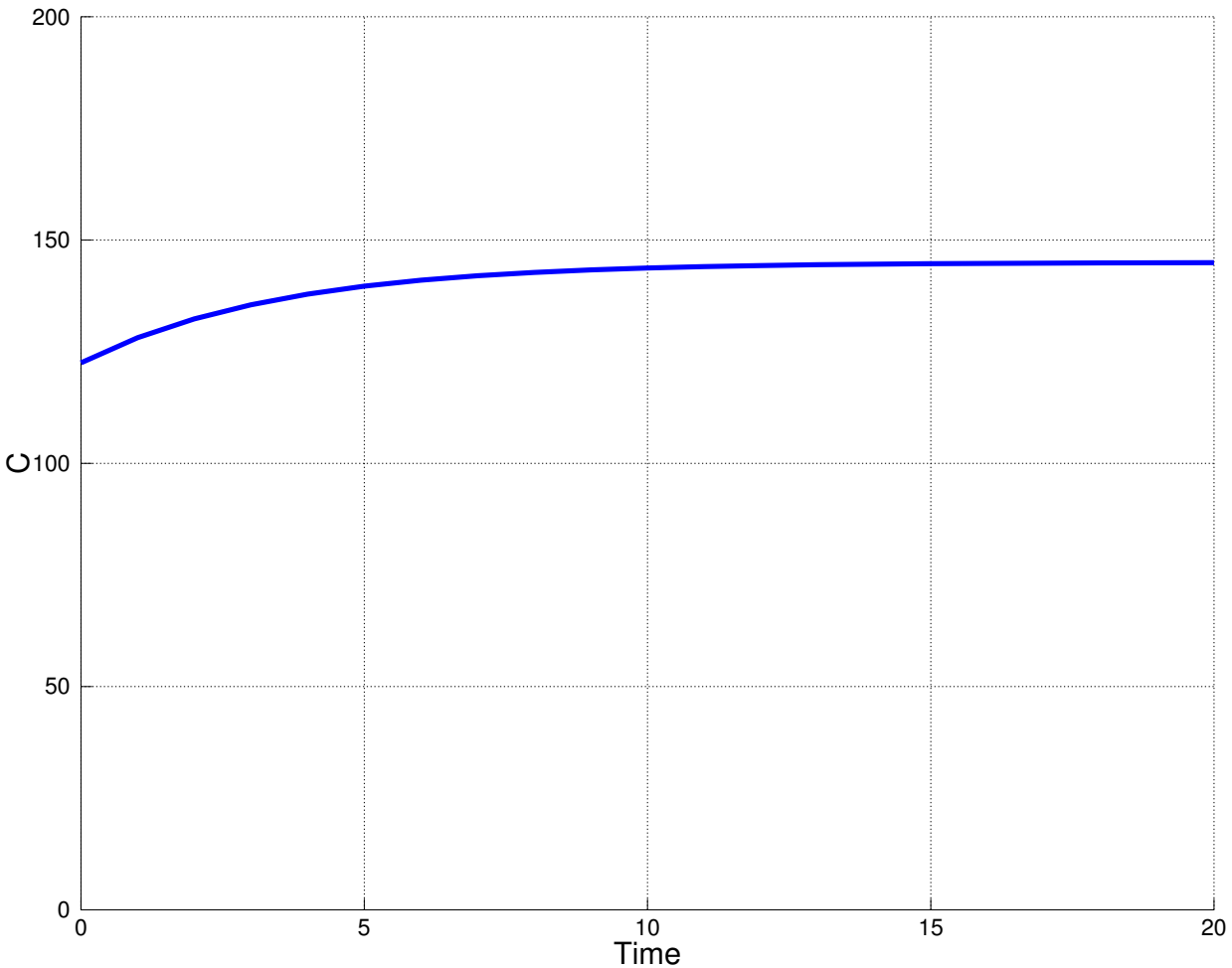
dEY, Y(1)=50



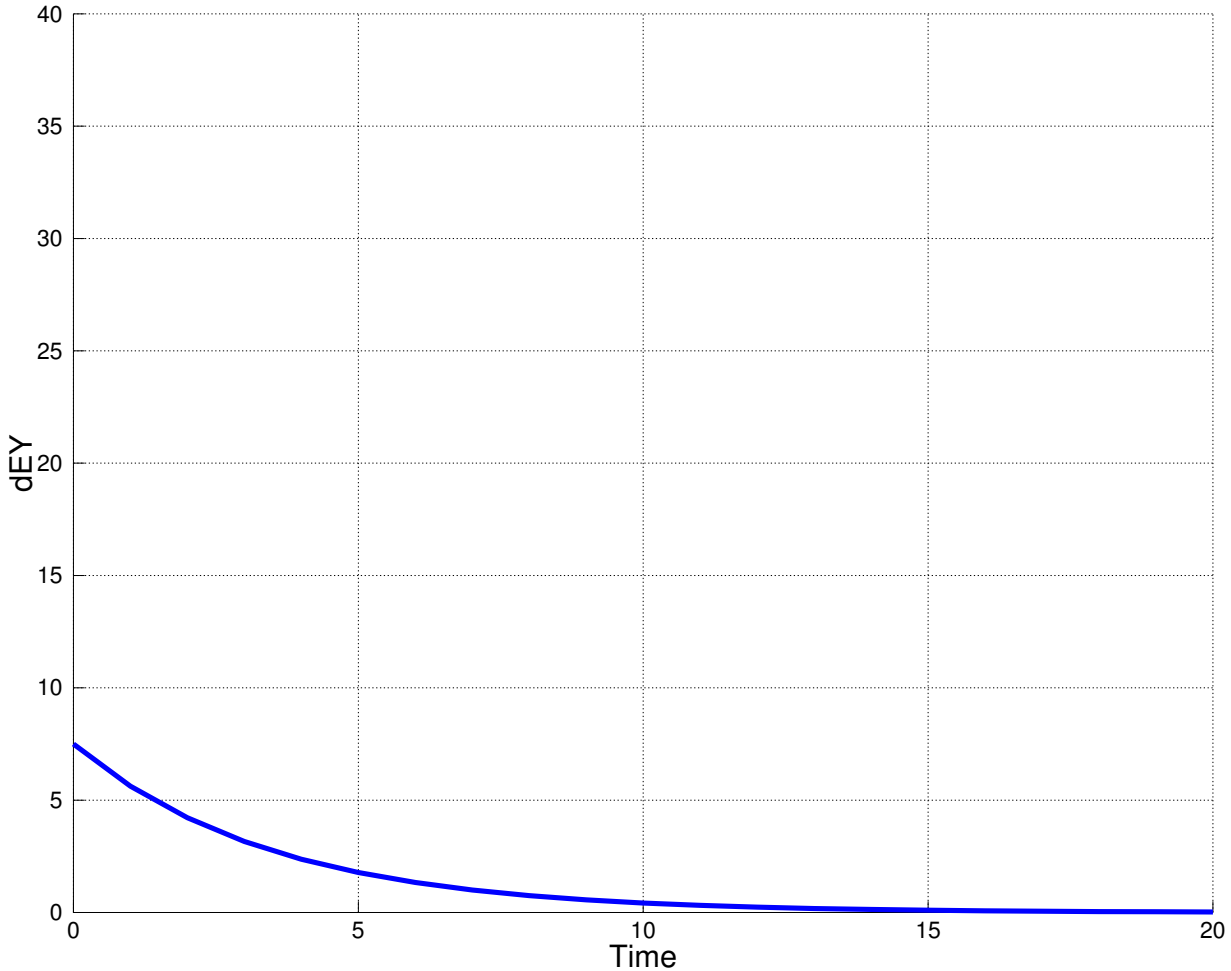
National Income, Y(1)=150



Consumption Expenditure, Y(1)=150



dEY, Y(1)=150



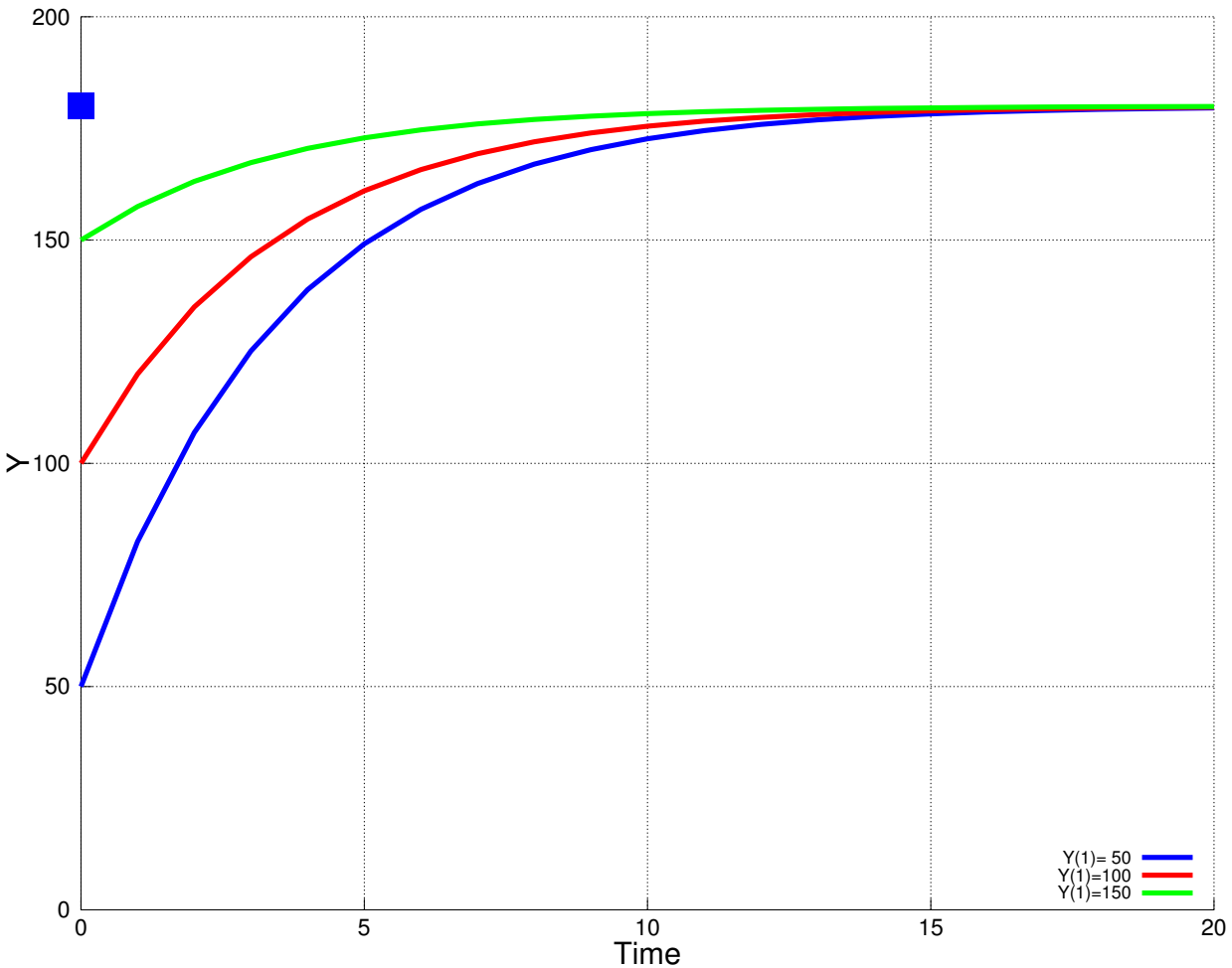
Save and plot the data

```
A = [t Y C E dEY];
save -ascii Keynes150.dat A;
```

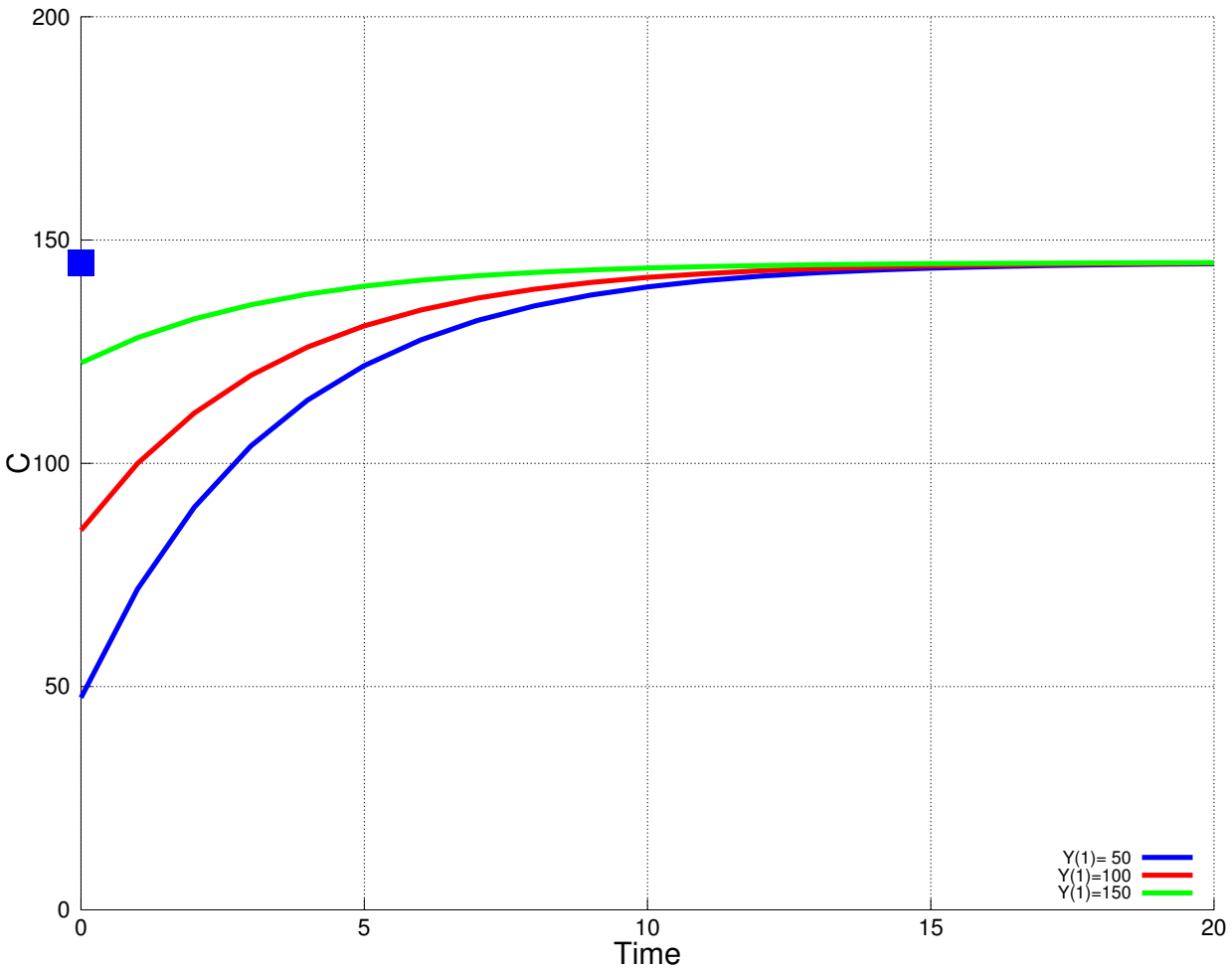
```
load Keynes100.dat;
load Keynes050.dat;
load Keynes150.dat;

plot(Keynes050(:,1), Keynes050(:,2), 'b', 'LineWidth', 8);
box off;
grid on;
xlabel('Time', 'FontSize', 24)
ylabel('Y', 'FontSize', 24)
axis([0 20 0 200]);
hold on;
plot(Keynes100(:,1), Keynes100(:,2), 'r', 'LineWidth', 8);
plot(Keynes150(:,1), Keynes150(:,2), 'g', 'LineWidth', 8);
legend ('Y(1)= 50', 'Y(1)=100', 'Y(1)=150', 'Location', 'SouthEast');
plot(0, 180, 's', 'MarkerSize', 20);
hold off;
```

KeynesPlot1.m

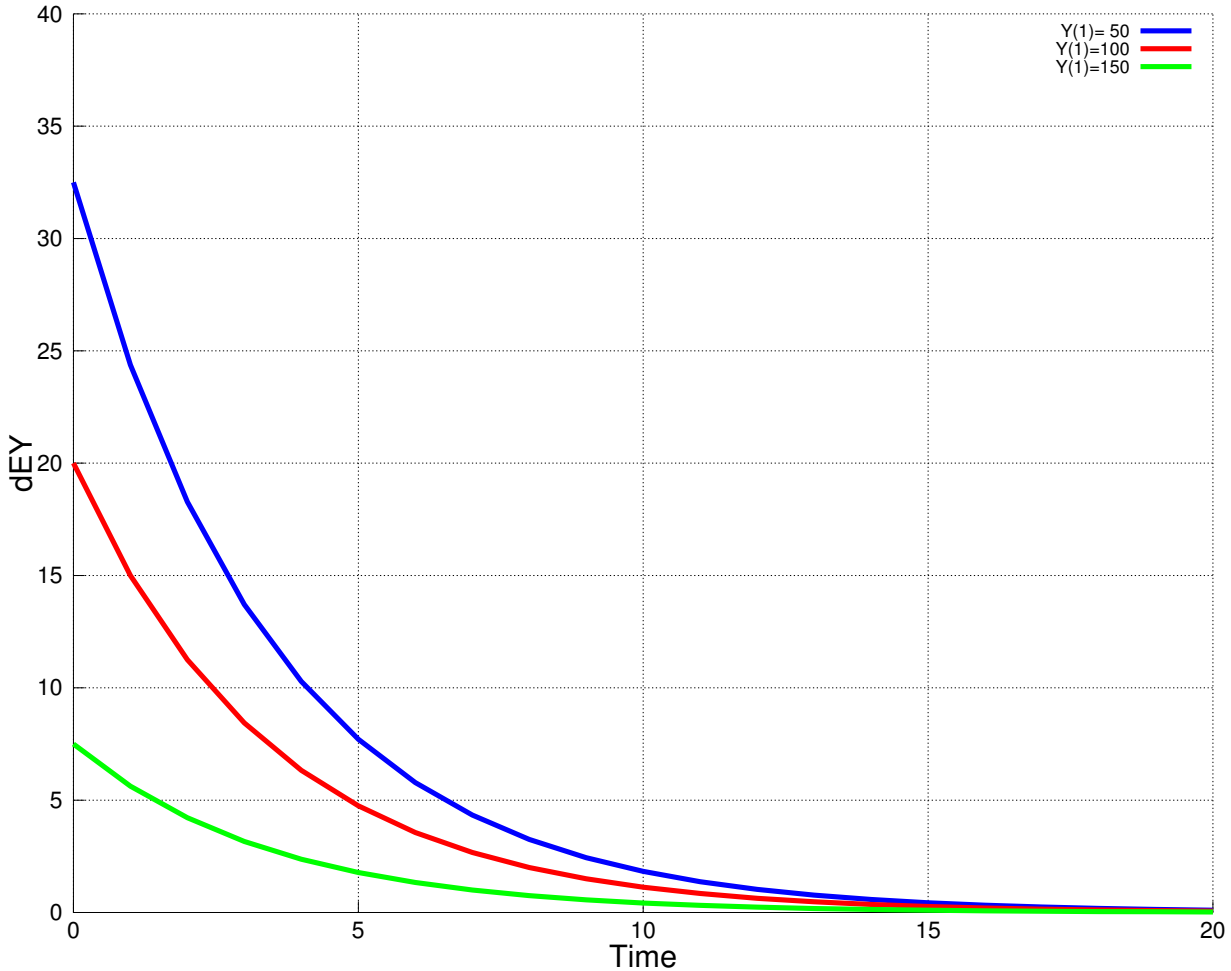
National Income, comparison



Consumption Expenditure, comparison



dEY, comparison



Importance of Y(0) in Keynes model

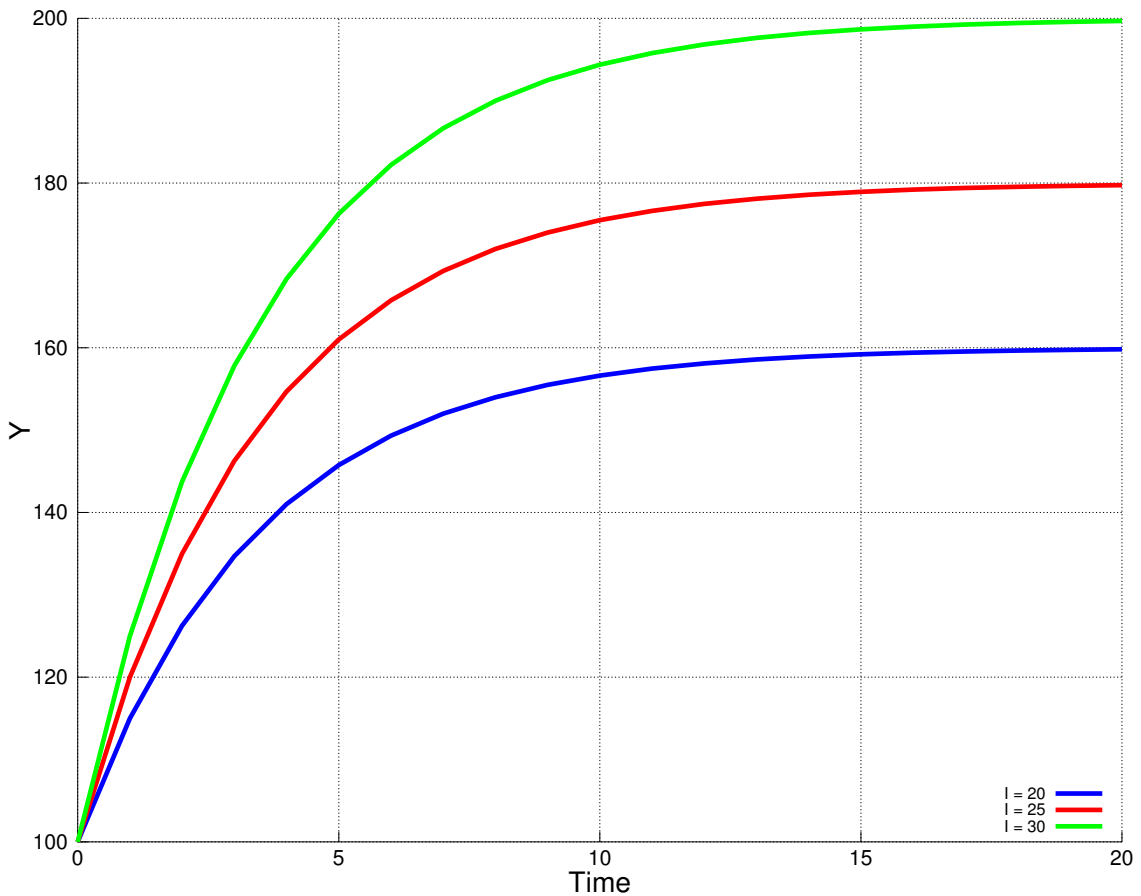
Equilibrium

$$
\begin{aligned}
C &= a + b\,Y \\
E &= C + I + G \\
Y &= E \\
E &= a + b\,Y + I + G \\
Y &= a + b\,Y + I + G \\
Y^* &= \frac{a+I+G}{1-b}
\end{aligned}
$$

Initial value

- Y^* is an attractor.
- $Y(0)$ only determines the rate of convergence.

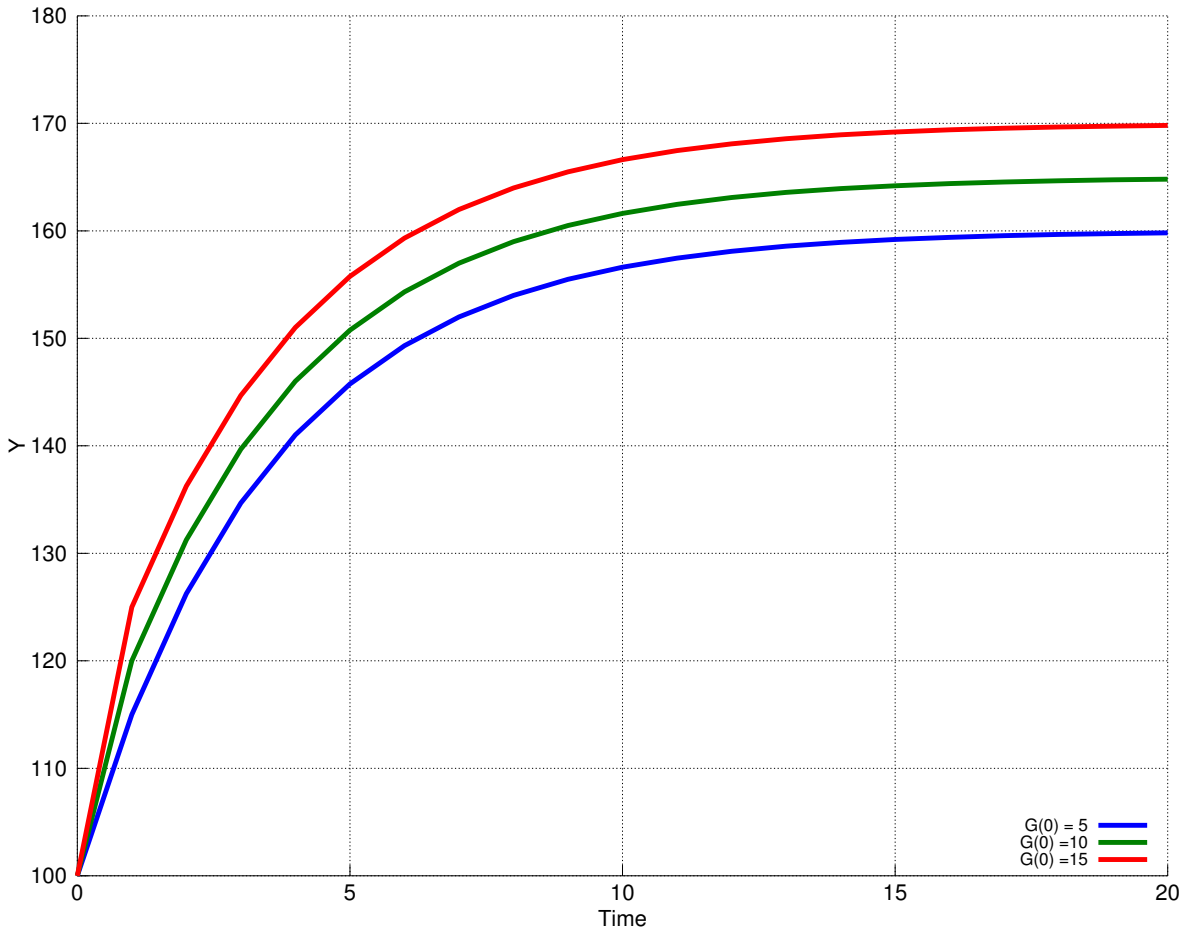
What about Investment?



Keynes2.m

KeynesPlot2.m

What about Government Expenditure?



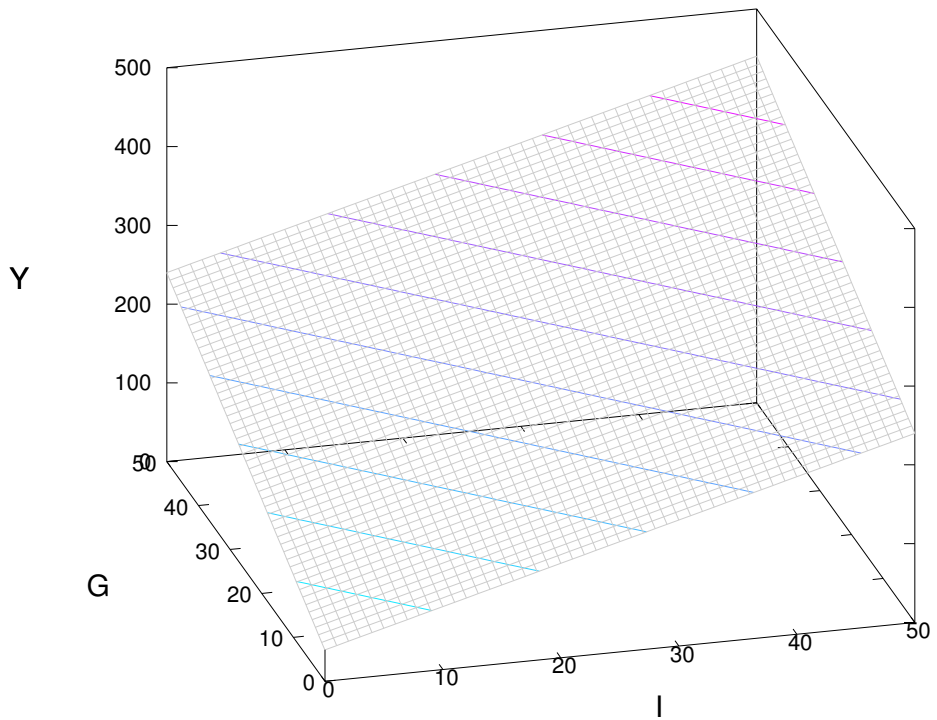
Keynes3.m

Use matrixes

```
a      = 10;
b      = 0.75;
I      = 25;
G      = [5 10 15];
lambda = 1;
T      = 20;
Y      = zeros(T+1, 3);
Y(1,:) = 100;

for (t=1:T)
  Y(t+1,:) =  lambda * (a+I+G) + (1-lambda*(1-b)) * Y(t);
end

t = (0:T)';
plot(t, Y, 'LineWidth', 8);
box off;
grid on;
axis([0 T 100 180]);
xlabel ('Time');
ylabel ('Y');
legend('G(0) = 5', 'G(0) =10', 'G(0) =15', 'Location', 'SouthEast');
```

Keynes3.m

I, G surface

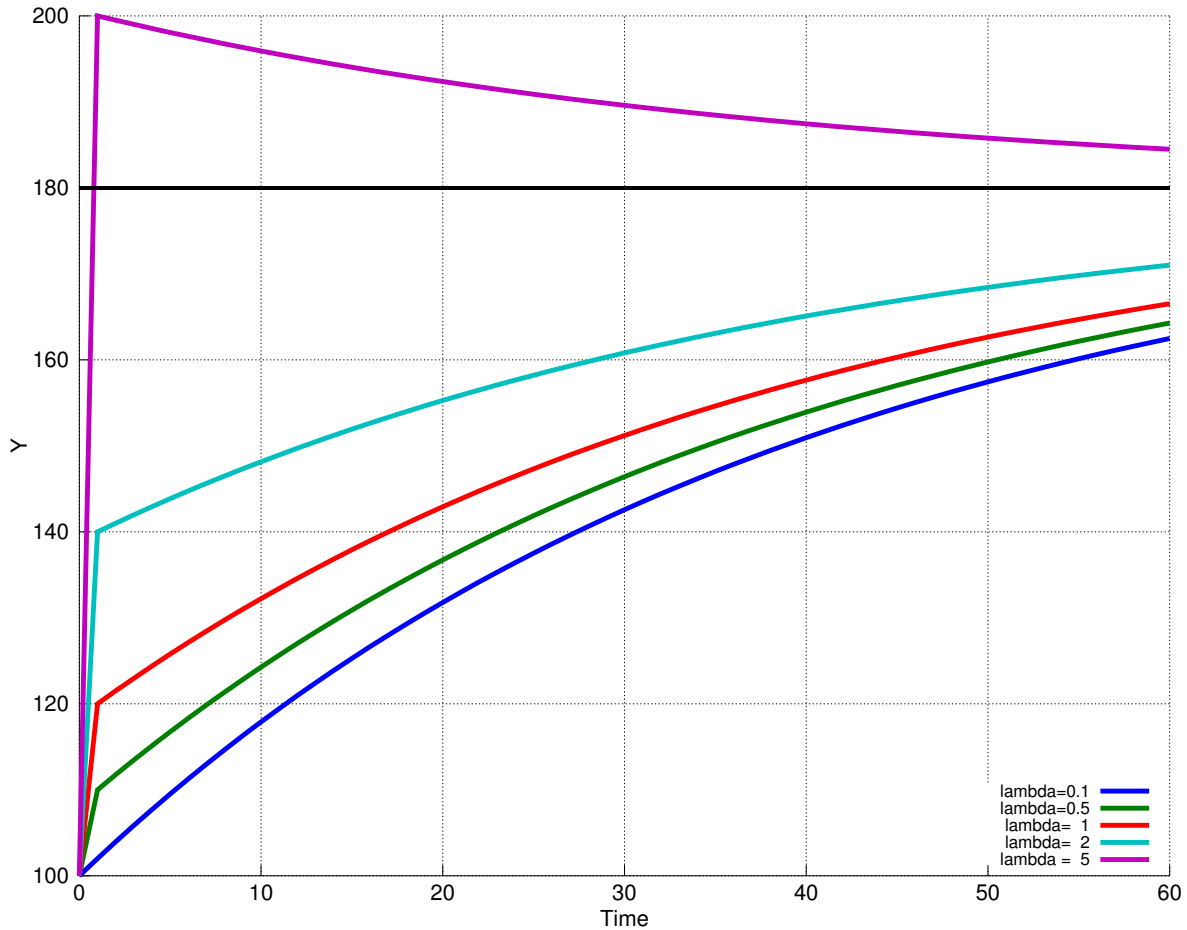


KeynesIGcontour.m

Surface plot

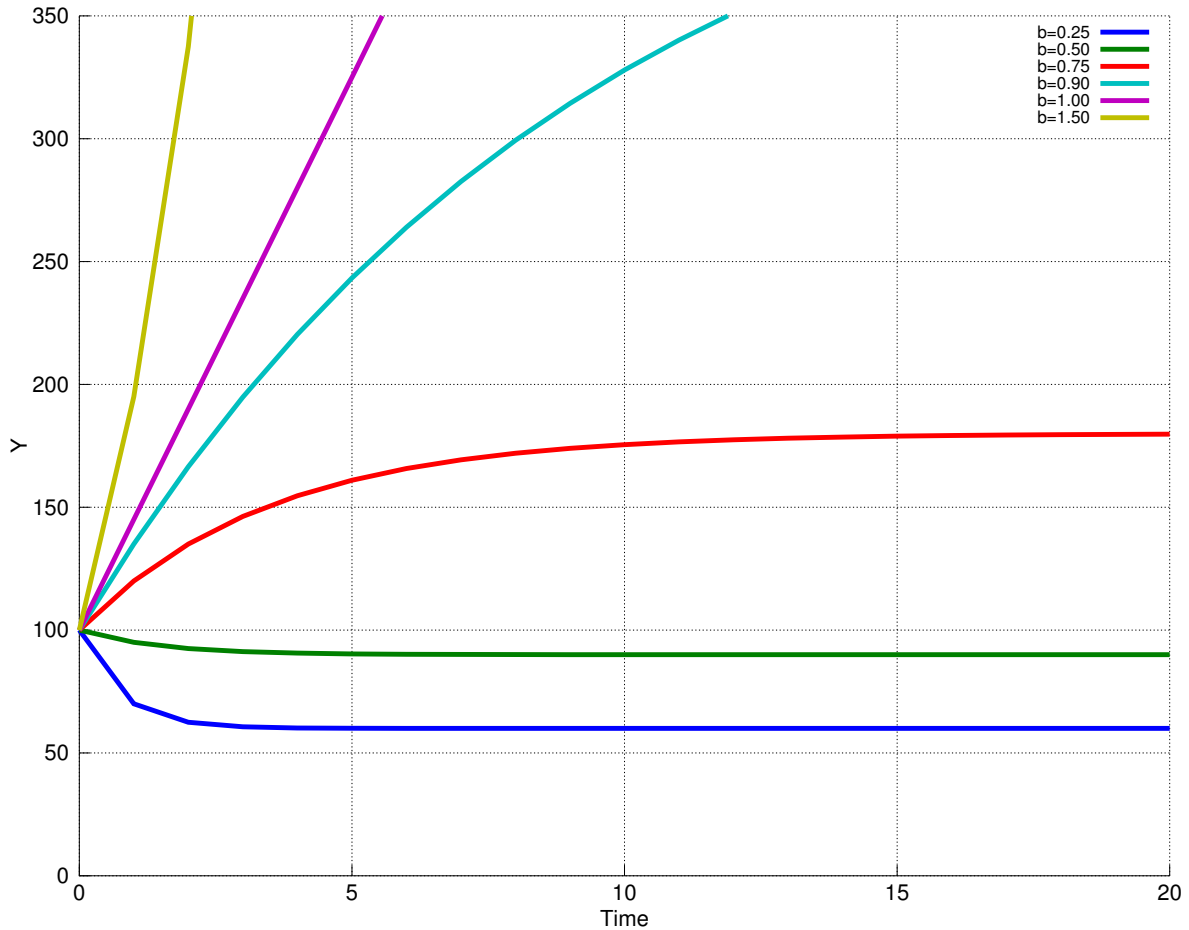
```
a = 10;
b = 0.75;

[I, G] = meshgrid(0:1:50, 0:1:50);

Y =  (a+I+G) / (1-b);
contour3(I, G, Y);
surface(I, G, Y, 'EdgeColor', [0.8 0.8 0.8], 'FaceCo
xlabel('I', 'FontSize', 24);
ylabel('G', 'FontSize', 24);
zlabel('Y', 'FontSize', 24);
grid off
view(-15, 30)
```

What about lambda?



Keynes4.m

What about b?



Keynes5.m

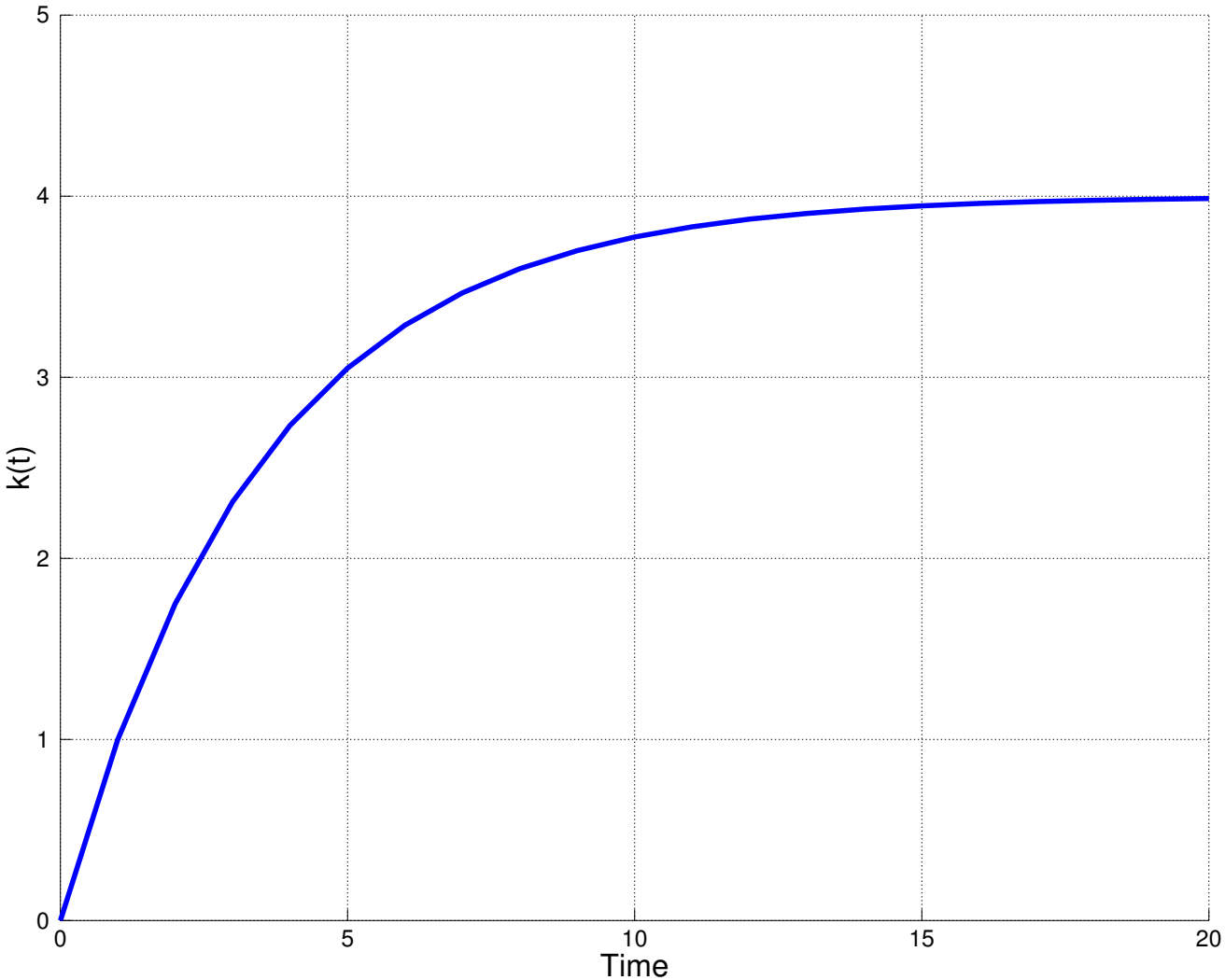
Attention to b

Keynes Model

$$\begin{aligned} C &= a + b\,Y \\ E &= C + I + G \\ Y &= E \end{aligned}$$

- b = marginal propensity to consume.
- Not only determines the equilibrium Y^*.
- It also determines if equilibrium exists.
- Generally, $0 < b < 1$.
- $b > 1$, there is no equilibrium (fixed) point.

The dynamic (period) multiplier



$$k = \frac{\Delta Y}{\Delta I}$$

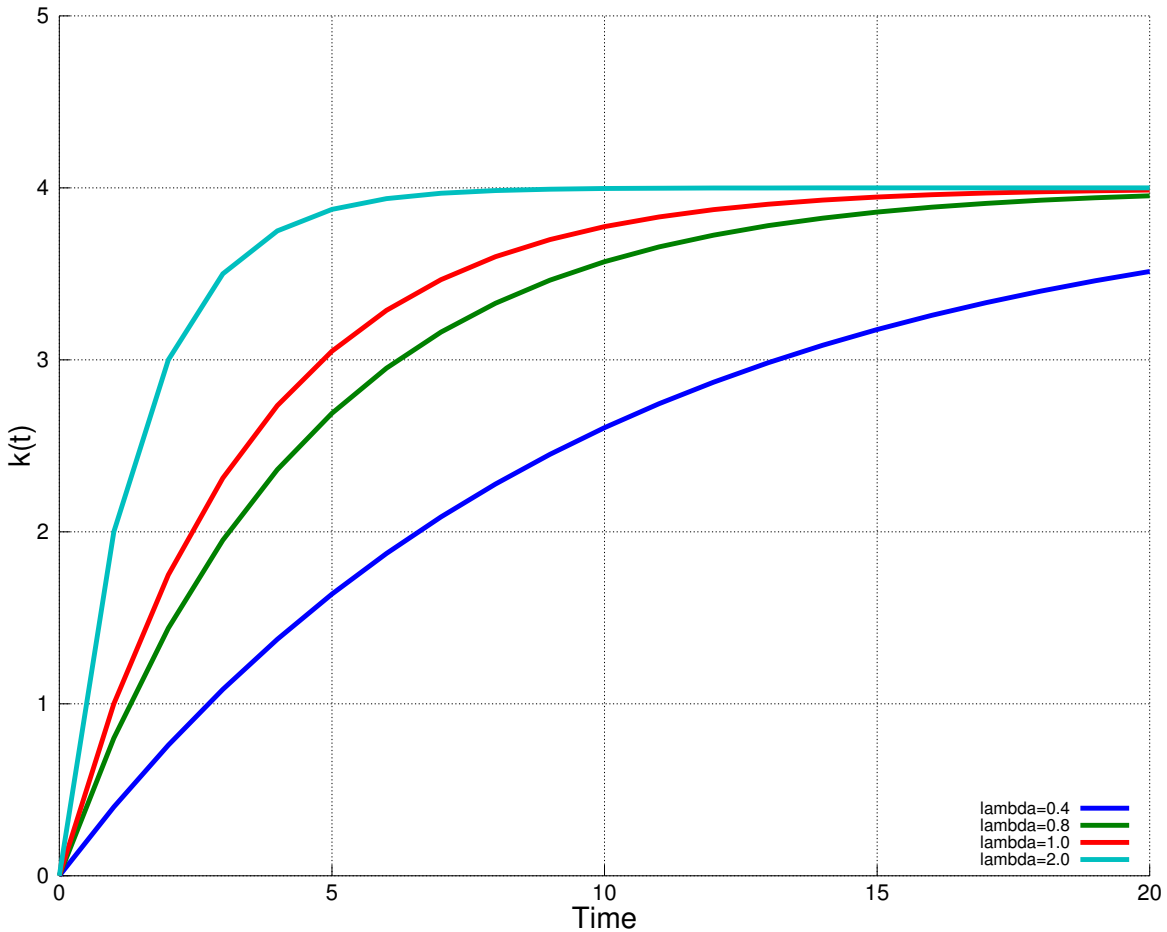
$$k_t = \frac{\Delta Y_t}{\Delta I} = \frac{Y_t - Y_1^*}{I_2 - I_1}$$

Multiplier1.m

Octave/Matlab code hint, Multiplier

```
1  a      = 10;
2  b      = 0.75;
3  I1     = 20;
4  I2     = 25;
5  G      = 10;
6  lambda = 1;
7  T      = 20;
8  Y      = zeros(T+1, 1);
9
10 Ystar1 = (a+I1+G) / (1-b);
11 Y(1)   = Ystar1;
12
13 for (t=1:T)
14   Y(t+1) =  lambda * (a+I2+G) + (1-lambda*(1-b)) * Y(t);
15 end
16 k = (Y-Ystar1) / (I2-I1);
```

How multiplier is influenced by lambda?



Multiplier2.m

Octave/Matlab code hint, Multiplier2

```
a      = 10;
b      = 0.75;
I1     = 20;
I2     = 25;
G      = 10;
lambda = [0.4  0.8  1  2];
T      = 20;
Y      = zeros(T+1, length(lambda));

Ystar1 = (a+I1+G) / (1-b);
Y(1,:) = Ystar1;

for (t=1:T)
  Y(t+1,:) = lambda*(a+I2+G)+(1-lambda*(1-b)) .* Y(t,:);
end
k = (Y - Ystar1) / (I2 - I1);
```

Taxation

Keynes model with taxes

$$
\begin{aligned}
C_t &= a + b\, Yd_t \\
Yd_t &= Y_t - tx\, Y_t \\
Tx_t &= Tx_0 + tx\, Y_t \\
E_t &= C_t + I + G \\
Y_t &= a + b\, Y_t + I + G \\
\Delta Y_{t+1} &= \lambda(E_t - Y_t) \quad , \quad \lambda > 0
\end{aligned}
$$

Notes

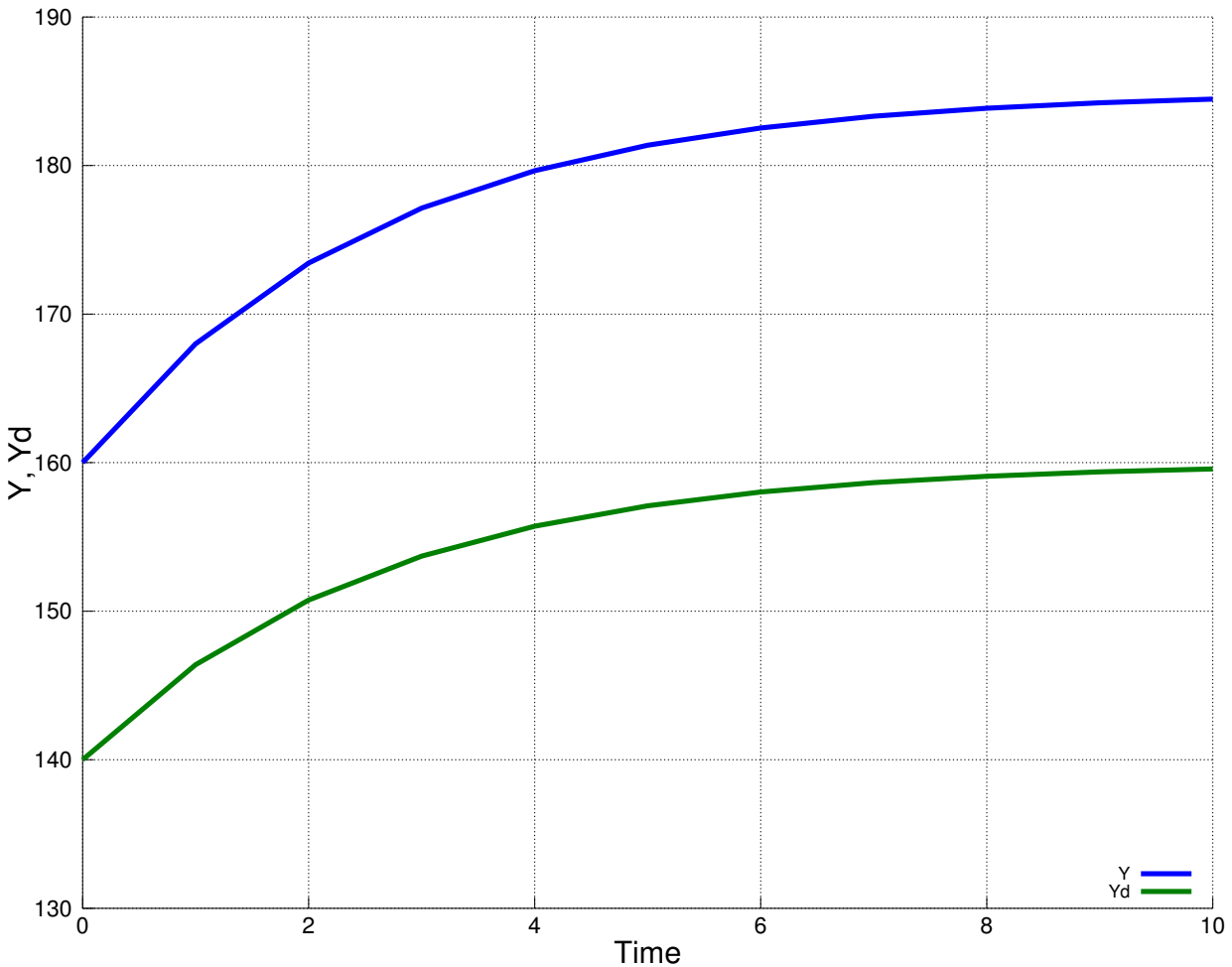
- Tx = total taxes
- tx = marginal rate of taxes
- t = time period
- I, G = exogenous variables

Taxation example

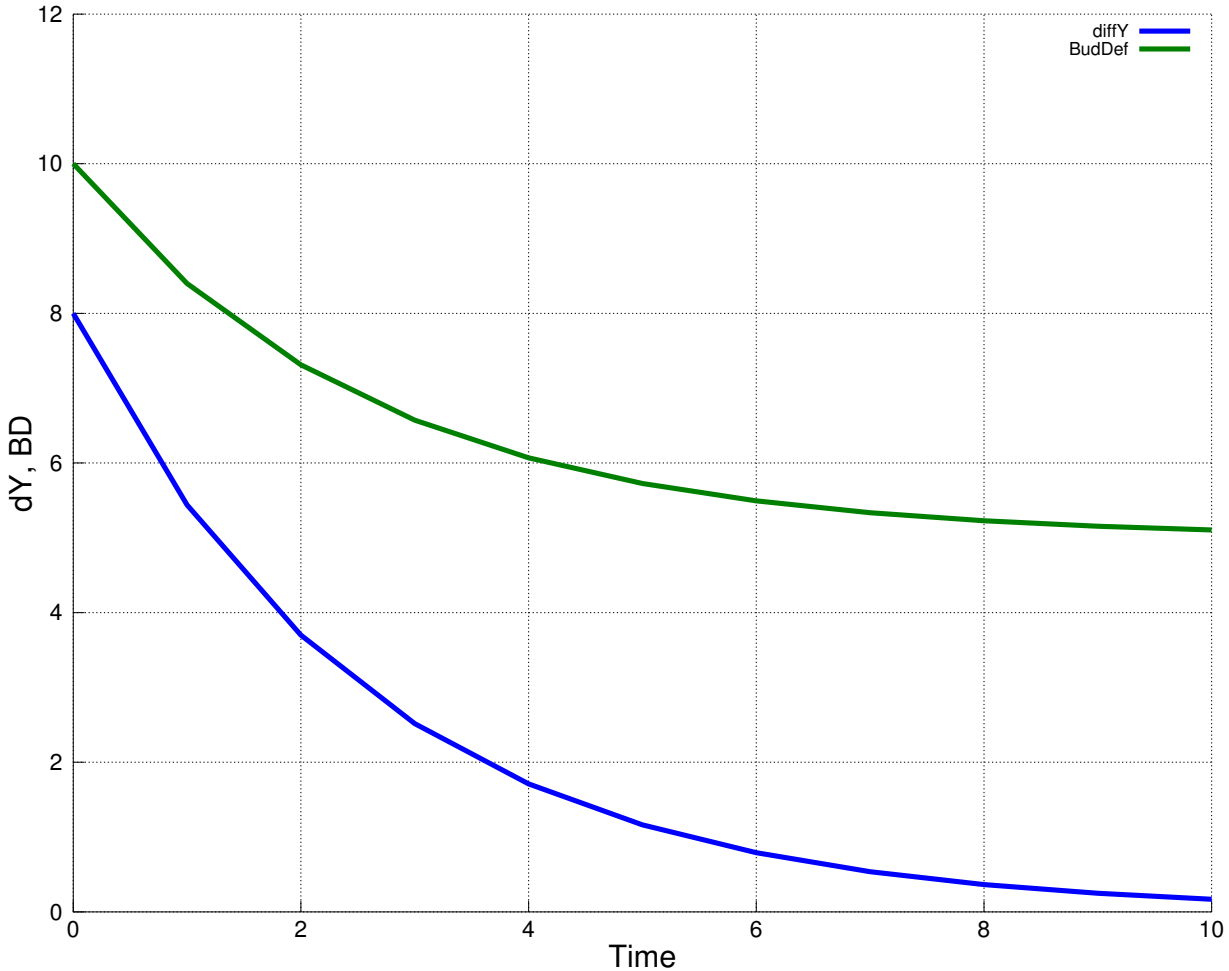
Example Keynes model with taxes

$$
\begin{aligned}
C_t &= 10 + 0.75\, Yd_t \\
Yd_t &= Y_t - 0.2\, Y_t \\
Tx_t &= -12 + 0.2\, Y_t \\
E_t &= C_t + 25 + 30 \\
Y_t &= 10 + 0.75\, Y_t + 25 + 30 \\
\Delta Y_{t+1} &= 0.8(E_t - Y_t)
\end{aligned}
$$

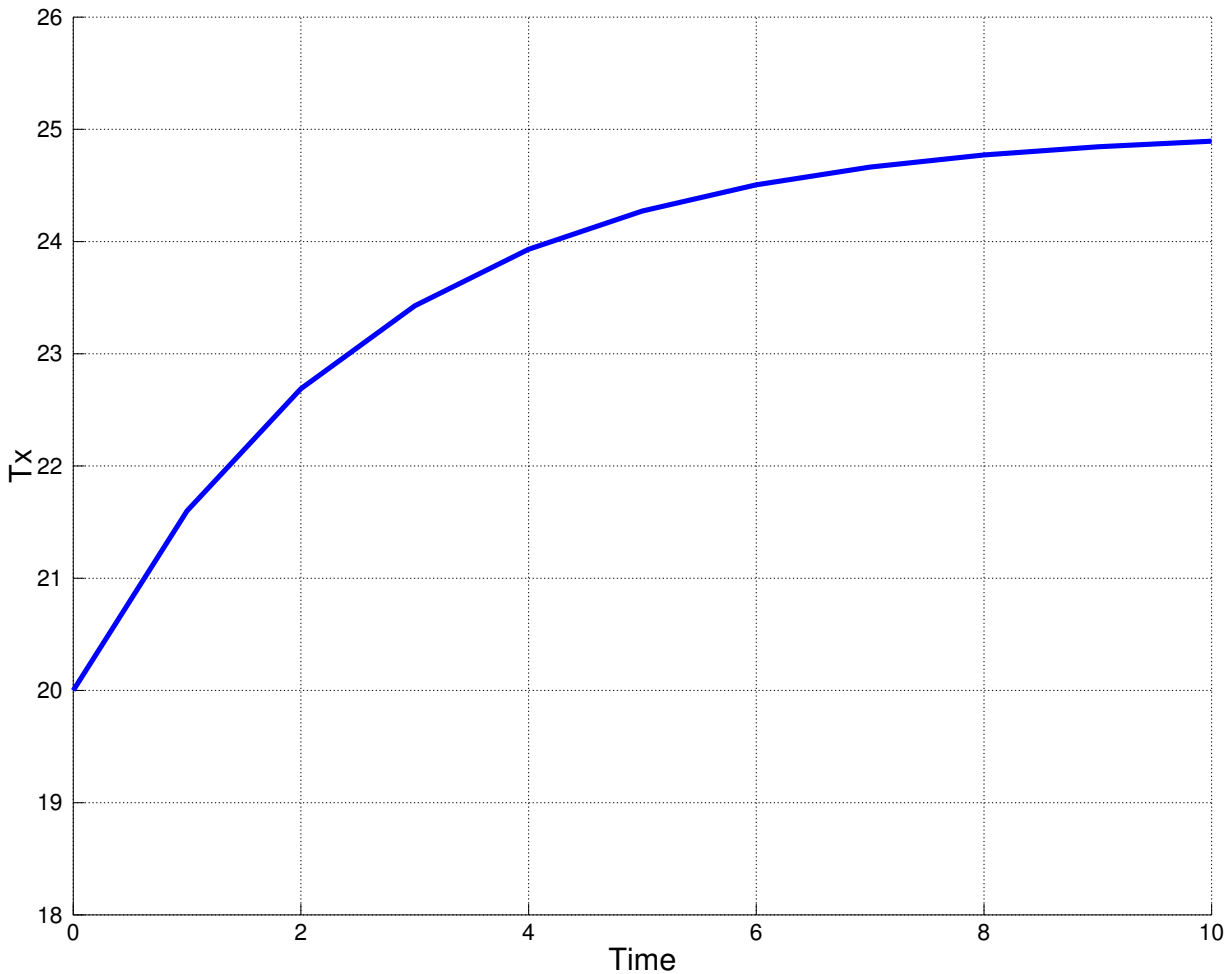
Income with taxation



Budget deficit



Total taxes



Octave/Matlab code hint, Taxation1

```
lambda = 0.8;
tx     = 0.2;
Tx0    = -12;

Ystar  = (a-b*Tx0+I+G) / (1-b*(1-tx));
Txstar = Tx0 + tx*Ystar;
DBstar = G - Txstar;
Gstar  = a + b*(Ystar-Txstar);

T      = 10;
Y      = zeros(T+1, 1);
Y(1)   = 160;

for (t=1:T)
  Y(t+1) = lambda * (a-b*Tx0+I+G) + ( 1-lambda*(1-b*(1-tx) ) ) * Y(t);
end

Tx     = Tx0 + tx*Y;      % taxes
Yd     = Y - Tx;          % disposable income
C      = a + b*Yd;        % consumption
E      = C + I + G;       % expenditure
DY     = lambda*(E-Y);    % diff income
BD     = G - Tx;          % budget deficit
```

Taxation1.m

Play with the taxation

- change $Y_0 = Y^*, 140, 210$
- change $tx = 0.3, 0$
- change $Tx_0 = -40, 0$
- change $G = 35, 20$
- comment the value of

$$\frac{1}{1 - b(1 - tx)}$$

The multiplier–accelerator model

Consumption

Consumption depends on previous period income:

$$C_t = a + b\, Y_{t-1}$$

Investment

Investment depends on the difference of income in the last two periods:

$$I_t = v(Y_{t-1} - Y_{t-2}) \quad , \quad \nu > 0$$

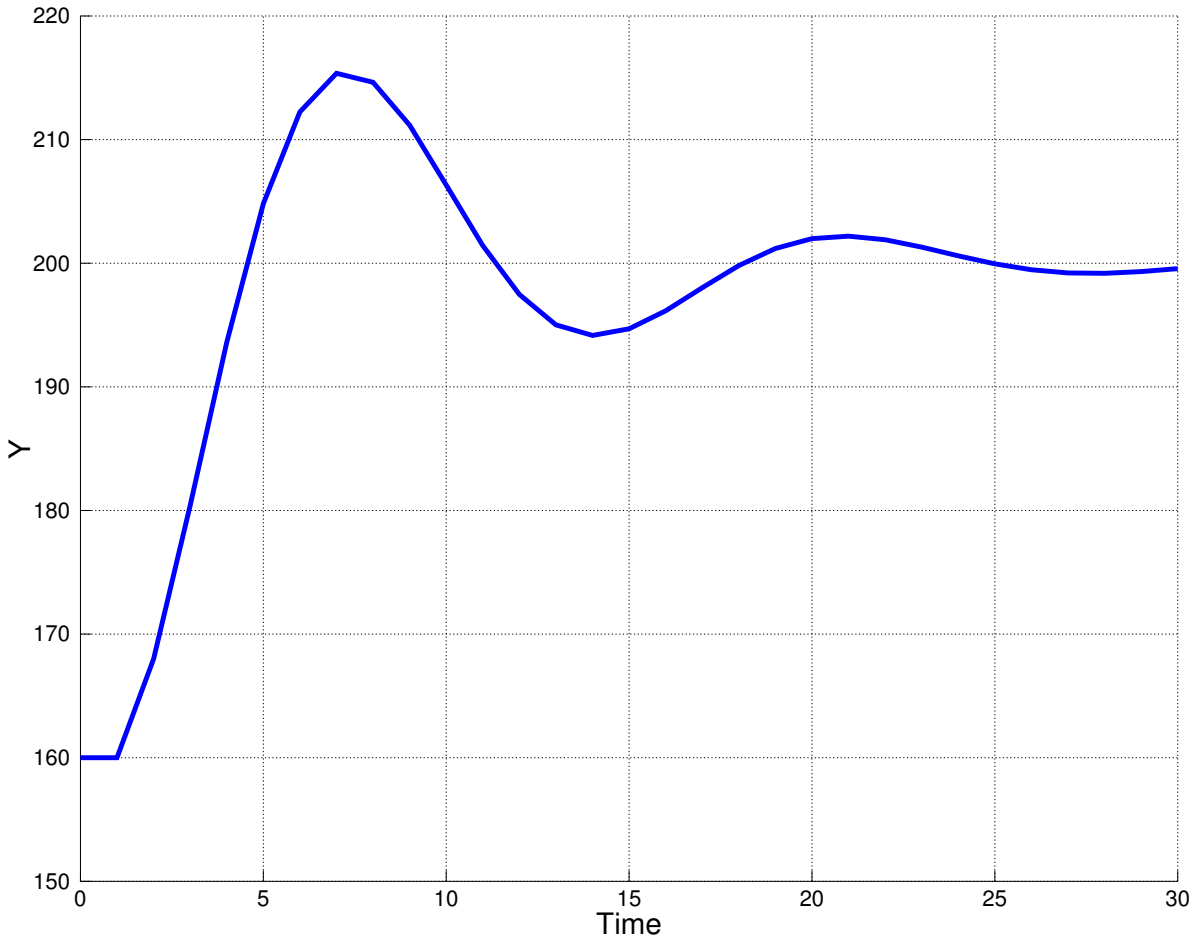
endogenous variables

Investment are no longer exogenous variables.

The multiplier–accelerator model

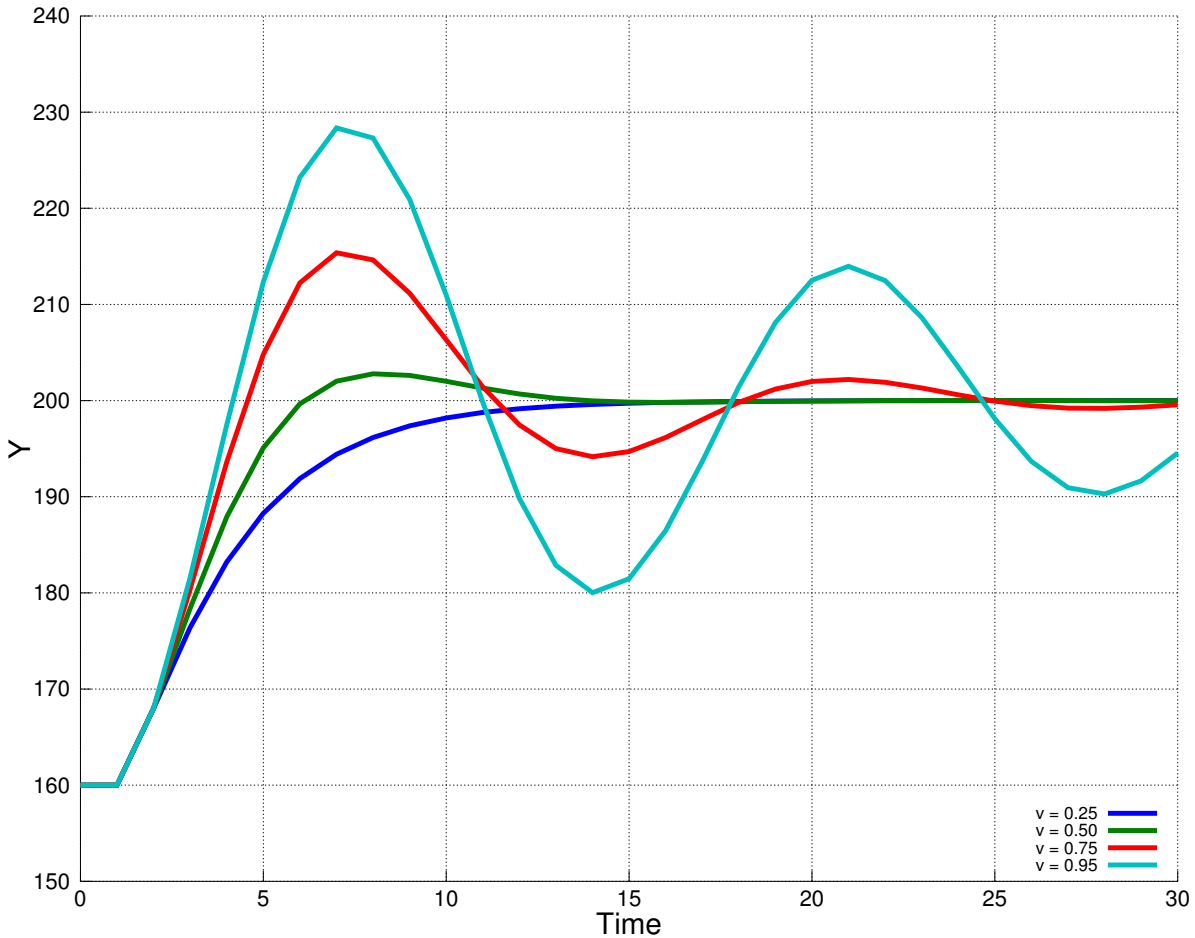
$$
\begin{aligned}
C_t &= a + b\, Y_{t-1} \\
I_t &= v(Y_{t-1} - Y_{t-2}) \\
E_t &= C_t + I_t + G \\
Y_t &= E_t \\
Y_t &= (a + G) + (b + v)\, Y_{t-1} - n\, Y_{t-2}
\end{aligned}
$$

Income vs Time, Keynes model with taxation



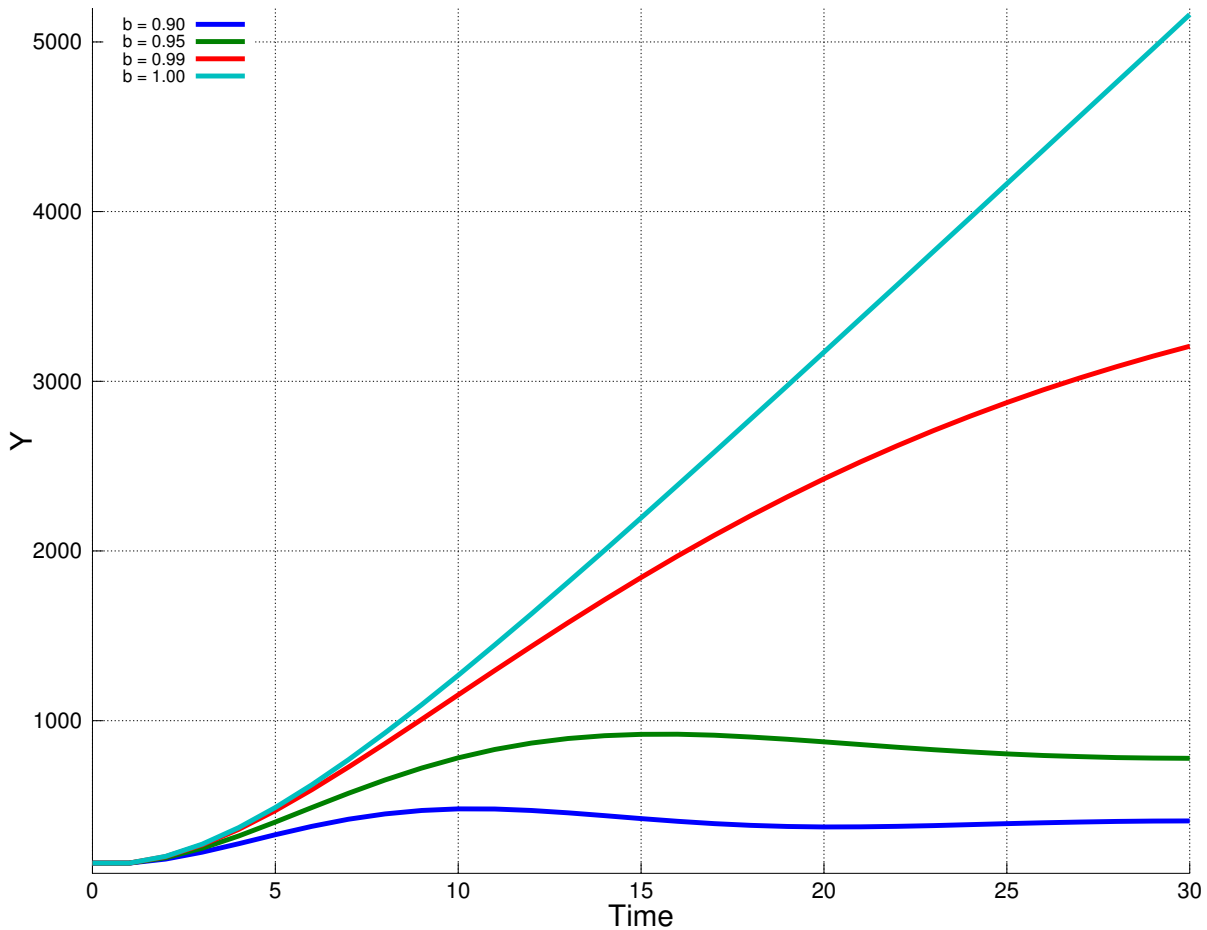
MultiAcce1.m

Influence of v



MultiAcce2.m

Influence of b



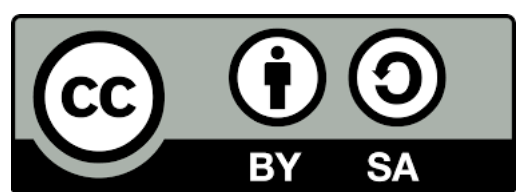
MultiAcce3.m

Σχόλια και ερωτήσεις

Σας ευχαριστώ για την προσοχή σας.

Είμαι στη διάθεσή σας για σχόλια, απορίες και ερωτήσεις.

Τέλος Ενότητας



Χρηματοδότηση

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ΕΣΠΑ 2007-2013
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ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.

Έχουν προηγηθεί οι κάτωθι εκδόσεις:

- Έκδοση 1.0 διαθέσιμη εδώ. http://ecourse.uoi.gr/course/view.php?id=1155.

Σημείωμα Αναφοράς

Σημείωμα Αδειοδότησης

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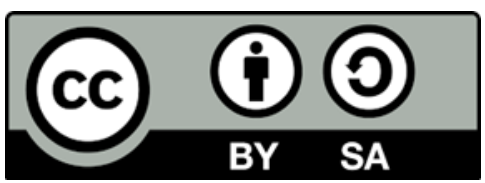

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