# ΜΕΡΙΚΕΣ ΠΑΡΑΓΩΓΟΙ

Για κάθε μια από τις παρακάτω συναρτήσεις να βρεθούν οι παράγωγοι, $f_x$, $f_y$ και $f_{xy}$

(1) $f(x,y) = x\sin(xy)$   $f_x = \sin(xy) + x\cos(xy)\,y$

$f_y = x^2\cos(xy)$   $f_{xy} = 2\,\cos(xy)\,x - x^2\sin(xy)\,y$

(2) $f(x,y) = x\cos\left(xy^2\right)$   $f_x = \cos\left(xy^2\right) - x\sin\left(xy^2\right)y^2$

$f_y = -2\,x^2\sin\left(xy^2\right)y$   $f_{xy} = -4\,\sin\left(xy^2\right)xy - 2\,x^2\cos\left(xy^2\right)y^3$

(3) $f(x,y) = ye^{xy}$   $f_x = y^2e^{xy}$

$f_y = e^{xy} + yxe^{xy}$   $f_{xy} = 2\,ye^{xy} + y^2xe^{xy}$

(4) $f(x,y) = yxe^{xy}$   $f_x = ye^{xy} + y^2xe^{xy}$

$f_y = xe^{xy} + yx^2e^{xy}$   $f_{xy} = e^{xy} + 3\,yxe^{xy} + y^2x^2e^{xy}$

(5) $f(x,y) = x^2e^{xy^2}$   $f_x = 2\,xe^{xy^2} + x^2y^2e^{xy^2}$

$f_y = 2\,x^3ye^{xy^2}$   $f_{xy} = 6\,x^2ye^{xy^2} + 2\,x^3y^3e^{xy^2}$

(6) $f(x,y) = \ln\left(x^2 - y^3\right)$   $f_x = 2\,\frac{x}{x^2 - y^3}$

$f_y = -3\,\frac{y^2}{x^2 - y^3}$   $f_{xy} = 6\,\frac{xy^2}{\left(x^2 - y^3\right)^2}$

(7) $f(x,y) = \ln\left(x^2 + y - y^3\right)$   $f_x = 2\,\frac{x}{x^2 + y - y^3}$

$f_y = \frac{1 - 3\,y^2}{x^2 + y - y^3}$   $f_{xy} = -2\,\frac{x\left(1 - 3\,y^2\right)}{\left(x^2 + y - y^3\right)^2}$

(8) $f(x,y) = \sin\left(x^2 - y^3\right)$   $f_x = 2\,\cos\left(x^2 - y^3\right)x$

$f_y = -3\,\cos\left(x^2 - y^3\right)y^2$   $f_{xy} = 6\,\sin\left(x^2 - y^3\right)y^2x$

(9) $f(x,y) = \sqrt{x^2 - y^3}$   $f_x = \frac{x}{\sqrt{x^2 - y^3}}$

$f_y = -3/2\,\frac{y^2}{\sqrt{x^2 - y^3}}$   $f_{xy} = 3/2\,\frac{xy^2}{\left(x^2 - y^3\right)^{3/2}}$

(10) $f(x,y) = \cos(xy)\,e^{xy}$   $f_x = (-\sin(xy) + \cos(xy))\,ye^{xy}$

$f_y = (-\sin(xy) + \cos(xy))\,xe^{xy}$   $f_{xy} = ((-1 - 2\,xy)\sin(xy) + \cos(xy))\,e^{xy}$

(11) $f(x,y) = \sin(x+y)\,e^{x-y}$   $f_x = \cos(x+y)\,e^{x-y} + \sin(x+y)\,e^{x-y}$

$f_y = \cos(x+y)\,e^{x-y} - \sin(x+y)\,e^{x-y}$   $f_{xy} = -2\,\sin(x+y)\,e^{x-y}$

(12) $f(x,y) = \sin(x+y)\,e^{xy}$   $f_x = \cos(x+y)\,e^{xy} + \sin(x+y)\,ye^{xy}$

$f_y = \cos(x+y)\,e^{xy} + \sin(x+y)\,xe^{xy}$   $f_{xy} = (\sin(x+y)\,yx + (x+y)\cos(x+y))\,e^{xy}$

# ΠΑΡΑΓΩΓΟΣ ΚΑΤΑ ΚΑΤΕΥΘΥΝΣΗ

Για κάθε συνάρτηση $f(x,y,z)$ να βρεθεί η παραγωγος στο σημειο $\vec{a}=(a,b,c)$ στην κατεύθυνση $\vec{u}=(u,v,w)$

| | $f(x,y,z)$<br>$\nabla f(x,y,z)$ | $\underline{a}=(a,b,c)$<br>$\nabla f(\underline{a})$ | $\underline{u}=(u,v,w)$<br>$\nabla f(\underline{a}) \bullet \underline{u}/\lvert\underline{u}\rvert$ |
|---|---|---|---|
| (1) | $xy^2z^3$<br>$[y^2z^3, 2\,xyz^3, 3\,xy^2z^2]$ | $(2,1,-1)$<br>$[-1,-4,6]$ | $(1,0,1)$<br>$5/\sqrt{2}$ |
| (2) | $xy^2+z^3$<br>$[y^2, 2\,xy, 3\,z^2]$ | $(2,1,-2)$<br>$(1,4,12)$ | $(1,-1,1)$<br>$3\sqrt{3}$ |
| (3) | $xy+\cos(z)$<br>$[y, x, -\sin(z)]$ | $(-1,1,\pi/2)$<br>$(1,-1,-1)$ | $(1,-1,1)$<br>$1/\sqrt{3}$ |
| (4) | $xz+ye^z$<br>$[z, e^z, x+ye^z]$ | $(1,1,0)$<br>$(0,1,2)$ | $(1,2,1)$<br>$2\sqrt{6}/3$ |
| (5) | $\cos(x)\,e^{yz}$<br>$[-\sin(x), \cos(x)\,z, \cos(x)\,y]e^{yz}$ | $(\pi,1,0)$<br>$(0,0,-1)$ | $(1,-1,1)$<br>$-1/\sqrt{3}$ |
| (6) | $\sin(x)\,e^{yz}$<br>$[\cos(x)\,e^{yz}, \sin(x)\,ze^{yz}, \sin(x)\,ye^{yz}]$ | $(\pi,1,0)$<br>$(-1,0,0)$ | $(1,-1,1)$<br>$-1/\sqrt{3}$ |
| (7) | $e^{x^3y^2z}$<br>$[3\,x^2y^2z, 2\,x^3yz, x^3y^2]e^{x^3y^2z}$ | $(-1,1,1)$<br>$(3,-2,-1)\,e^{-1}$ | $(1,-1,1)$<br>$4/\,e\sqrt{3}$ |
| (8) | $e^{x+y^2+z^3}$<br>$[e^{x+y^2+z^3}, 2\,ye^{x+y^2+z^3}, 3\,z^2e^{x+y^2+z^3}]$ | $(1,2,-1)$<br>$(e^4, 4\,e^4, 3\,e^4)$ | $(1,-2,-1)$<br>$-5e^4\sqrt{6}/3$ |
| (9) | $e^{xy^2+z^3}$<br>$[y^2, 2\,xy, 3\,z^2]e^{xy^2+z^3}$ | $(1,-1,-1)$<br>$(1,-2,3)$ | $(2,-2,-1)$<br>$1$ |
| (10) | $e^{xy^2\cos(z)}$<br>$[y^2\cos(z), 2\,xy\cos(z), -xy^2\sin(z)]e^{xy^2\cos(z)}$ | $(1,-1,\pi/2)$<br>$(0,0,-1)$ | $(2,-2,-1)$<br>$1/3$ |

# ΚΛΙΣΗ ΣΥΝΑΡΤΗΣΗΣ

Για κάθε συνάρτηση $f(x,y,z)$ να βρεθεί η κλίση της συνάρτησης $\nabla f(x,y,z)$ στο σημειο $\vec{a}=(a,b,c)$.

(1) $f(x,y,z) = xy + z^2$ $\quad (a,b,c) = (1,-1,2)$

$\nabla f(x,y,z) = [y, x, 2\,z]$ $\quad \nabla f(1,-1,2) = [-1,1,4]$

(2) $f(x,y,z) = xy^2z^3$ $\quad (a,b,c) = (3,2,-1)$

$\nabla f(x,y,z) = [y^2z^3, 2\,xyz^3, 3\,xy^2z^2]$ $\quad \nabla f(3,2,-1) = [-4,-12,36]$

(3) $f(x,y,z) = xy^2 + e^{zx}$ $\quad (a,b,c) = (1,-1,2)$

$\nabla f(x,y,z) = [y^2 + ze^{zx}, 2\,xy, xe^{zx}]$ $\quad \nabla f(1,-1,2) = [1+2\,e^2, -2, e^2]$

(4) $f(x,y,z) = xy^2 + \cos(zx)$ $\quad (a,b,c) = [1,-1,\pi]$

$\nabla f(x,y,z) = [y^2 - \sin(zx)\,z, 2\,xy, -\sin(zx)\,x]$ $\quad \nabla f(1,-1,\pi) = [1,-2,0]$

(5) $f(x,y,z) = z\sin(xy)$ $\quad (a,b,c) = \left(1,\frac{\pi}{2},2\right)$

$\nabla f(x,y,z) = [z\cos(xy)\,y, z\cos(xy)\,x, \sin(xy)]$ $\quad \nabla f\left(1,\frac{\pi}{2},2\right) = [0,0,1]$

(6) $f(x,y,z) = z\cos(xy)$ $\quad (a,b,c) = \left(1,\frac{\pi}{2},2\right)$

$\nabla f(x,y,z) = [-z\sin(xy)\,y, -z\sin(xy)\,x, \cos(xy)]$ $\quad \nabla f\left(1,\frac{\pi}{2},2\right) = [-\pi,-2,0]$

(7) $f(x,y,z) = \sin(xyz)$ $\quad (a,b,c) = \left(1,\frac{\pi}{2},2\right)$

$\nabla f(x,y,z) = [\cos(xyz)\,yz, \cos(xyz)\,zx, \cos(xyz)\,xy]$ $\quad \nabla f\left(1,\frac{\pi}{2},2\right) = \left(-\pi,-2,-\frac{\pi}{2}\right)$

(8) $f(x,y,z) = x\sqrt{y^2+z^3}$ $\quad (a,b,c) = (1,2,-1)$

$\nabla f(x,y,z) = \left[\sqrt{y^2+z^3}, \frac{xy}{\sqrt{y^2+z^3}}, \frac{3xz^2}{2\sqrt{y^2+z^3}}\right]$ $\quad \nabla f\,(1,2,-1) = \left(\sqrt{3}, \frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{2}\right)$

(9) $f(x,y,z) = -\cos(xyz)$ $\quad (a,b,c) = \left(1,\frac{\pi}{2},-1\right)$

$\nabla f(x,y,z) = [\sin(xyz)\,yz, \sin(xyz)\,zx, \sin(xyz)\,xy]$ $\quad \nabla f\left(1,\frac{\pi}{2},-1\right) = \left[\frac{\pi}{2},1,-\frac{\pi}{2}\right]$